Soliton Absorption Spectroscopy in Normal-Dispersion Lasers

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Abstract  An all-normal-dispersion oscillator with narrowband intracavity absorption acquires spectral features which follow the index of refraction of the absorber, as confirmed by experimental observation. Compared to the conventional absorption, the signal can be enhanced by an order of magnitude.

Introduction

Fiber-optic sensors have a number of applications in a variety of fields such as environment and industrial monitoring including trace gas detection. One of possible implementations is the intracavity absorption (ICA). The ICA is of special interest for gas spectroscopy because the broad spectra of fiber lasers (e.g. Nd, Er, and Tm-doped fibers) cover the overtone absorption lines of a number of important gases. The ICA can be introduced artificially for spectroscopic measurements or wavelength calibration, like acetylene lines in Er-fiber lasers, but can also occur from atmospheric absorption in a holey fiber or free-air path of a resonator. For example, water vapour lines provide up to 2% absorption per 1 cm path in the Tm-fiber laser range. It has been recently shown, that in a conventional soliton oscillator, ICA results in a spectral modulation that follows the associated index of refraction of the absorber.

Modelocked oscillators operating in the all-normal dispersion (ANDi) regime possess interesting features, one of them being the linearization of the nonlinear phase shift, which allows increasing of the spectral bandwidth, without destabilizing the pulse. By tailoring the core-clad index profile of a fiber or by designing a high index-contrast micro-structured fiber to shift the zero-dispersion towards longer wavelength, one can extend this technology towards Er- and Tm-fiber lasers. Besides their robustness, such oscillators would allow broad spectra with almost flat-top profile, which is attractive for measurements.

In this work we present a quantitative analysis of a chirped dissipative soliton perturbed by narrowband absorption and demonstrate the feasibility of soliton absorption spectroscopy in an ANDi oscillator. The signal can be increased by about an order of magnitude in comparison to conventional absorption over the same path length. Besides scientific interest, this result also paves the way towards a novel conceptually simple and robust fiber optic based instrumentation that is capable of real-time broadband absorption measurements of a wide range of gases.

Model

The model of an ANDi oscillator is based on the perturbed complex cubic-quintic nonlinear Ginzburg-Landau equation:

\[
\begin{align*}
\dot{a} &= g a + \left(\alpha - i \frac{D}{2}\right) a a_t + \left(\kappa + i \gamma\right) |a|^2 a - \\
&- \kappa \zeta |a|^4 a + \Gamma |a|,
\end{align*}
\]

where the field envelope \(a\) depends on the propagation distance \(z\) and the local time \(t\), \(D\) is the group-velocity dispersion, \(\gamma\) is the self-phase modulation parameter. The dissipative factors of an oscillator are described by: the net saturated gain \(g\), the spectral filtering (\(\alpha\ being the square of the inverse bandwidth), and the self-amplitude modulation \(\kappa\), which is saturable (the parameter \(\zeta\)). The perturbation operator \(\Gamma |a|\) describes the narrowband absorption in an oscillator (e.g. a gas mixture), which can be described in the frequency domain as a superposition of \(l\) causal Lorentz lines with the absorption coefficients \(\epsilon_l\) and the linewidths \(\Omega_l\) (the signs before \(i\) in Eq. (1) and the following expression correspond to the fiber optics convention):

\[
\Gamma |\tilde{a}| = \left[ \sum_l \frac{\epsilon_l}{1 - i(\omega - \omega_l)/\Omega_l} \right] \tilde{a}.
\]

In the normal dispersion range \((D>0)\), the unperturbed Eq. (1) has a dissipative soliton (DS) solution with a strong chirp. Such a DS is stable within a broad range of parameters and describes the pulse generated by an ANDi oscillator. The approximate solution of (1) with \(\Gamma = 0\) for the DS complex spectral amplitude is:

\[
\tilde{a} (\omega) \approx \sqrt{\frac{6 \pi \gamma}{\kappa \zeta (\omega^2 + \Theta)}} (\Delta^2 - \omega^2) \times \\
\times \exp \left[ \frac{3 i \gamma^2 \omega^2}{\kappa \zeta (\omega^2 + \Theta)} \right],
\]

which results from the assumption, that the DS is strongly chirped and, therefore, the stationary phase method for the Fourier transformation

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can be applied. Here \( H (\Delta^2 - \omega^2) \) is the Heaviside step function, and \( \Delta \), \( \Theta \) are the parameters defining the spectrum half-width and the shape of its top \(^5\). The spectrum (3) has a Lorentzian (or parabolic) profile truncated at \( \pm \Delta \).

**Perturbative analysis of DS spectrum**

The solution (3) in combination with the assumption of a strong chirp allows performing perturbative analysis in the Fourier domain. Since \( \gamma \gg \kappa, \zeta, D \gg \alpha \), and \( q \approx 0^{14,12} \), we may neglect the contribution of dissipative terms into the equation for the DS perturbation \( f(t) \exp [iqz] \ (q = -\Delta^2 D/2 \) is the DS wave number):

\[
-\imath q f = -\frac{iD}{2} f_{tt} + \imath \gamma \left[ 2 |a|^2 f + a^2 f^* \right] + \tilde{\Gamma} [a], \quad (4)
\]

In the spectral domain, the last equation results in

\[
\left[ k(\omega) + q \right] \tilde{f} + \frac{1}{\pi} \int \limits_{-\infty}^{\infty} d\omega' U(\omega - \omega') \tilde{f}(\omega') + \frac{1}{2\pi} \int \limits_{-\infty}^{\infty} d\omega' V(\omega - \omega') \tilde{f}^*(\omega') = S(\omega), \quad (5)
\]

where \( U \) and \( V \) are defined through the Fourier images of \(|a|^2\) and \( a^2 \), respectively. These images can be obtained by means of the method of stationary phase \(^8\). The source of perturbation is \( S(\omega) = i\gamma \left[ \tilde{f} \right] \) and the linear wave number is \( k(\omega) = \omega^2 D/2 \).

The conjecture \( V \otimes \tilde{f}^* = U \otimes \tilde{f} \) allows solving the integral equation (5) in the form of Neumann series:

\[
\tilde{f}_n = \frac{S}{k + q} - \frac{3}{2\pi(k + q)} \times \int \limits_{-\infty}^{\infty} d\omega' U(\omega - \omega') \tilde{f}_{n-1}(\omega'), \quad (6)
\]

with \( \tilde{f}_0 = S/(k + q) \).

For the zero-order term of (6) and under assumptions of \(|\epsilon| \ll 1\) (weak absorption) and \( \Omega_l \ll \Delta \) (narrow absorption lines), one has

\[
\frac{\tilde{P}}{\tilde{P}_0} \approx 1 + \frac{4}{D} \times \sum \limits_{l} \frac{\epsilon_l}{(\Delta^2 - \omega_l^2)^{1/2}} \left( \frac{\omega_l - \omega_l}{\Omega_l} \right)^2, \quad (7)
\]

In this case the spectrum is flat-top and \( \alpha \Delta^2 = 3gc/(c - 2) \), where \( c \equiv 2\alpha\gamma/D\kappa \).

**Results**

The obtained expressions (6,7) for the DS spectral profile in the presence of narrowband absorption have the interesting characteristics.

1) The spectral power \( \tilde{P} \) follows the associated index of refraction of the absorber (Fig. 1). Locally, within a narrow spectral range in the vicinity of an absorption line, \( \tilde{P} \) is perfectly matched by the zero-order term of (6) (circles vs. dashed curve in Fig. 1) and does not depend on the higher-order dissipative nonlinearities (solid curve vs. circles in Fig. 1).

2) In the vicinity of spectrum centre \((\omega = 0)\), the amplitude of perturbation is \( 2\epsilon/\omega_l \) (Eq. (7)), same as in the case of the perturbed Schrödinger soliton \(^4\). Decrease of \(|\eta|\) (Fig. 2) enhances the spectral amplitude of perturbation and enhances the sensitivity of ICA spectroscopy. An enhancement grows with \( D/\alpha \) (i.e. with higher dispersion or the wider filter band) as well as with the lower dimensionless energy \( E \), normalized to \( \gamma/\sqrt{\alpha} \), see Fig. 2. On the other hand, both energy decrease and dispersion growth would narrow the DS spectrum. For instance, a 15-fold enhancement of ICA signal can be achieved at \( D \approx 0.018 \) ps\(^2\) and \( E \approx 0.15 \) nJ, narrowing the DS spectrum to \( \approx 5 \) nm for \( \kappa = 0.04\pi \) and other parameters of Fig. 2.

3) In contrast to the Schrödinger soliton \(^4\), the amplitude of the DS perturbation grows when \( \omega_l \) moves away from the spectrum centre, see Eq. (7) and Fig. 3. The enhancement diverges in the vicinity of spectrum edges at \( \omega = \pm \Delta \).
Fig. 2: Dependence of the DS wave-number $q$ on the dimensionless energy $E$ and the relative dispersion $D/\alpha$. For the $\alpha$-parameter corresponding to the 27 nm bandwidth and $\gamma=0.015$ W$^{-1}$, $E=10^4$ corresponds to $\approx 14$ nJ and $D/\alpha=100$ corresponds to $D=0.045$ ps$^2$. $\kappa=0.01\gamma$, $\delta=\ell^2/g_0=0.1$, where $\ell$ is the net-loss, $g_0$ is the gain for a small signal.

4) Besides the dispersion-like signature, the narrowband absorption also creates weak anti-symmetric sidelobes, which extend much further beyond the absorption line (Fig. 5). This is confirmed by an experiment using a chirped-pulse Cr:ZnSe laser operating at open air.

Conclusions
The perturbative analysis of the chirped dissipative soliton in an ANDi fiber laser with narrowband absorption shows, that the pulse remains stable, but its spectrum traces the effective refractive index of absorber. The spectral signatures are substantially enhanced in comparison with conventional spectroscopy spectroscopy. This makes the soliton absorption spectroscopy a very attractive technique for spectroscopy of gases and impurities, environment monitoring, and metrology.

Fig. 3: DS spectrum under action of 5 symmetrically placed absorption lines ($f_0$ approximation) with $\Omega_l=4$ GHz and $\epsilon_l=0.005$. Other parameters are: $\alpha=450$ fs$^2$, $D=200\alpha$, $\kappa=0.01\gamma$, $\zeta=0.2\gamma$, $g=-0.0016$.

Fig. 4: DS spectrum under action of one absorption line: $f_0$ approximation (dashed curve) and $f_1$ approximation (solid curve) with $\Omega_l=4$ GHz and $\epsilon_l=0.005$. Other parameters are: $\alpha=450$ fs$^2$, $D=200\alpha$, $\kappa=0.01\gamma$, $\zeta=0$, $g=-0.0016$.

Fig. 5: High-resolution spectrum of a chirped-pulse Kerr-lens modelocked Cr:ZnSe laser. Only a narrow part near the water vapour absorption line is shown.

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References
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