Stochastic Calculations for Fibre Raman Amplifiers with Randomly Varying Birefringence (I)

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For the model of real-world forward-pumped fibre Raman amplifier with randomly varying birefringence [1-3], the stochastic calculations have been done numerically based on the Kloeden-Platen-Schurz algorithm (see [4] for physical and mathematical details).

Refs.:

The parameters are defined by a fibre length $L$ (in meters), a correlation length $L_c$ defining stochastic birefringence of a fibre, beat lengths for a pump $p$ and a signal $s$: $L_{bp}$ and $L_{bs}$, a Raman gain coefficient $g$ (in $1/(W\cdot km)$), and an input pump power $P$ (in W). $a_s$ and $a_p$ are the loss coefficients for signal and pump, respectively (0.48 and 0.5 dB/km in the case considered). $\eta=1$ corresponds to copropagating pulse and signal.
\( \eta = 1; \)
\( L = 5000; \)
\( Lc = \frac{100}{L}; \) (*normalizations to fibre length*)
\( Lbs = \frac{20}{L}; \)
\( Lbp = Lbs \times 1.55 / 1.465; \) (*signal and pump wavelengths in micrometers*)
\( Lmin = \text{Min}[Lc, Lbs, Lbp]; \)
\( \text{step} = 10^{-4} \times Lmin; \)
\( bs = \frac{\pi}{Lbs}; \)
\( bp = \frac{\pi}{Lbp}; \)
\( g = 0.8; \)
\( gn = g \times L / 1000; \)
\( \text{Pin} = 1; \)
\( \text{as} = \text{Solve}[\text{Power}[10^{-0.48}, (10)^{-1}] == \text{Exp}[-x], x][[1]][[1]][[2]]; \)
\( \text{ap} = \text{Solve}[\text{Power}[10^{-0.5}, (10)^{-1}] == \text{Exp}[-x], x][[1]][[1]][[2]]; \)
\( P0[z_] := \text{Pin} \times \text{Exp}\left[-\text{ap} \times z \times L / 1000\right]; \)
(*pump decay with propagation due to loss*)

The Stokes representation for states of polarisation (SOP) is used but a "scalar part" of Raman gain is excluded by the normalization [1,2]:

\[
\text{s0in} = \text{Exp}\left[\int_0^{L/1000} (g \text{Pin} \text{Exp}[-\text{ap} \times z] / 2 - \text{as} \times z) \, dz\right];
\]

Stokes parameters for a signal are defined by a power \( s0 \) (input power is 10 mW in this example) and by unit vector \((s1, s2, s3)\). Pump SOP is defined by an unit vector \((p1, p2, p3)\). \( x \) is a scalar product of pump and signal SOPs. Stochastic birefringence affecting the local birefringence vector angle \( \phi \) is defined by the Wiener process with volatility \( =1/Lc \). The system of eight stochastic differential equations defines an evolution (stochastic trajectories) of parameters [4]. Ten trajectories for \( x \) are under consideration in our case.
The mean value of projection scalar $x$ and its standard deviation are shown below.

```math
\begin{align*}
sys &= \{ \text{ds0[z]} &= (1/2) \ast \text{gn} \ast \text{P0[z]} \ast x[z] \ast s0[z] \ast \text{dz}, \\
s0[z] \ast ds1[z] + s1[z] \ast ds0[z] &= (1/2) \ast \text{gn} \ast \text{P0[z]} \ast p1[z] \ast s0[z] \ast \text{dz} + \\
s2[z] \ast s0[z] \ast \text{d} \beta[z], s0[z] \ast ds2[z] + s2[z] \ast ds0[z] &= \\
\text{dz} \ast (1/2) \ast \text{gn} \ast \text{P0[z]} \ast s0[z] \ast p2[z] - 2 \ast bs \ast s3[z] \ast s0[z] \} + \\
s1[z] \ast s0[z] \ast \text{d} \beta[z], s0[z] \ast ds3[z] + s3[z] \ast ds0[z] &= \\
\text{dz} \ast (2 \ast bs \ast s2[z] \ast s0[z] + (1/2) \ast \text{gn} \ast \text{P0[z]} \ast p3[z] \ast s0[z])
\end{align*}
```

```math
\begin{align*}
dp1[z] &= \frac{(p2[z] \ast \text{d} \beta[z])}{\eta}, \\
dp2[z] &= \frac{((-p1[z]) \ast \text{d} \beta[z] - 2 \ast bp \ast p3[z] \ast \text{d}z)}{\eta}, \\
dp3[z] &= \frac{(2 \ast bp \ast p2[z] \ast \text{d}z)}{\eta}, \\
s0[z] \ast dx[z] + x[z] \ast ds0[z] &= (\text{dz} \ast ((1/2) \ast \text{gn} \ast \text{P0[z]} \ast s0[z] - \\
2 \ast (bp \ast \eta - bs) \ast (p3[z] \ast s2[z] \ast s0[z] - p2[z] \ast s3[z] \ast s0[z])) + \\
(1 - \eta) \ast \text{d} \beta[z] \ast \{p1[z] \ast s2[z] \ast s0[z] - p2[z] \ast s1[z] \ast s0[z]\} + \\
\text{d} \phi[z] &= \frac{\text{d} \beta[z]}{\eta}
\end{align*}
```

IC = \{0.01, 1, 0, 0, 1, 0, 0, 1, 0\};

proc = StratonovichProcess[sys, x[z]],

\{\{s0, s1, s2, s3, p1, p2, x, \phi\}, IC\}, \eta \propto WienerProcess[0, 1/Lc]]; 

path = RandomFunction[proc, \{0., 1., step\}, 10, 

Method \rightarrow "KloedenPlatenSchurz"]; 

mf[z_] = Mean[path[z]]; 

dsf[z_] = StandardDeviation[path[z]]; 

pf[z_] = path[z];
```

```
pl = Plot[mf[z], \{z, 0, 1\}, PlotStyle \rightarrow \{Red, Dashed, Thick\}]

(* mean value of scalar product *)
```

![Graph of stochastic polarization](stochastic_polarisation.cdf)
The Fourier transforms and power spectra of x evolution are shown below.
FFTValues = Abs[Fourier[RotateLeft[data, 1/step/2 - 1], FourierParameters -> {-1, 1}]];

ListLinePlot[10*Log10[RotateRight[FFTValues, 1/step/2 - 1]],
PlotRange -> All, Filling -> Axis, Ticks ->
{N[Table[{i, i*step - 0.500008}, {i, 1, 1/step, 1/step/5}], Automatic]}

ListLinePlot[10*Log10[RotateRight[FFTValues, 1/step/2 - 1]],
PlotRange -> {{1/step/2 - 50, 1/step/2 + 50}, {-30, 0}},
Filling -> Axis, Ticks ->
{N[Table[{i, i*step - 0.500008}, {i, 1, 1/step, 1/step/5}], Automatic]]
The corresponding correlation function looks like:

```math
ListLinePlot[CorrelationFunction[data, {1/step - 1}], Filling -> Axis, PlotRange -> All, Ticks -> {N[Table[{i, i*step}, {i, 1, 1/step, 1/step/5}]], Automatic}]
```

And the histogram demonstrating an escape from metastable state [5] is

```
Histogram[data]
```
Lbs-parameter scanning reveals the phenomenon of resonant stochastization, so-called stochastic anti resonance. Despite a well-known effect of noise suppression and global regularization of dynamics due to resonant interaction of noise and regular external periodic perturbation, as it takes a place in the case of stochastic resonance [6-8], here we can observe a reversed situation when regular perturbation assists a noise-induced escape of a system from metastable state.