The unified theory of chirped-pulse oscillators

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Motivation and problem definition

- **Requirements**
  - over-μJ pulse energy
  - sub-100 fs width
  - MHz repetition rate

- **Aims**
  - high-harmonics generation
  - frequency combs
  - micro-machining

- **NDR vs. PDR**
  - NDR: i) dispersion scales as $E^2$; width of linearly non-compressible pulse $\propto E$
  - PDR: i) strong chirp damps the dispersion scaling; ii) pulse is linearly compressible

- **Solid-state* vs. fiber** **CPOs**
  - a ANDi fiber oscillator as opposed to a solid-state CPO has:
    - i) large dispersion;
    - ii) narrow gain-band
    - iii) large self-phase modulation

Mechanisms of pulse formation I: Soliton

Negative GDD

SPM
Mechanisms of pulse formation II: CSP

Phase balance

\[ \beta \frac{\partial^2}{\partial t^2} a(t) - \beta a(t) \left( \frac{\partial}{\partial t} \phi(t) \right)^2 \]

Phase balance: \( \psi^2 = 2 + \frac{\gamma P_0}{\beta T^2} \)

Change of envelope:

SAM is required!
Mechanisms of pulse formation III: CSP

Pulse shortening

Wigner functions

soliton

chirped pulse

spectral cutoff

time profiles
Three main types of CPOs

- **Broad-band solid-state oscillators (Ti:sapphire, Cr:YAG, Cr:ZnSe)**
  - broad gainband, i.e. weak spectral dissipation (inverse gainbandwidth ≤ 10 fs)
  - low GDD (~100 fs²)
  - moderate SPM (5÷20 MW⁻¹)
  - medium energy (100 nJ ÷ 1 µJ)

- **Narrow-band solid-state oscillators (Yb-doped thin-disk as an active medium)**
  - narrow gainband (inverse gainbandwidth 50÷100 fs)
  - medium GDD (tens of fs²)
  - low SPM (0.1÷2.5 GW⁻¹)
  - large energy (>10 µJ)

- **ANDi fiber oscillators**
  - narrow gainband (like thin-disk oscillator)
  - large GDD (~0.1 ps²)
  - large SPM (~10 kW⁻¹)
  - low or medium energy (few tens ÷ few hundreds of nanojouls)
Model and principal factors

Generalized complex nonlinear Ginzburg-Landau equation

\[
\frac{\partial a(z,t)}{\partial z} = i \left\{ \beta \frac{\partial^2}{\partial t^2} \left[ \gamma + \chi P(t) \right] P(t) \right\} a(z,t) + \left( -\sigma(E) + \alpha \frac{\partial^2}{\partial t^2} + X(P) \right) a(z,t)
\]

generalized nonlinear Schrödinger part
generalized nonlinear Ginzburg-Landau part

- \( a(z,t) \) is the slowly varying laser field profile
- \( P = |a|^2 \) is the power
- \( \beta \) is the net group-delay dispersion (GDD)
- \( \gamma \) is the self-phase modulation (SPM)
- \( \chi \) is the quintic self-phase modulation
- \( \sigma \) is the net saturable loss depending on the field energy \( E \)
- \( \alpha \) is the squared inverse gainband width
- \( X \) is the real function of power describing the self-amplitude modulation (SAM)
Types of nonlinearity

Cubic complex nonlinear Ginzburg-Landau equation

\[ \frac{\partial a(z,t)}{\partial z} = i \left\{ \beta \frac{\partial^2}{\partial t^2} - \gamma P(t) \right\} a(z,t) + \left( -\sigma(E) + \alpha \frac{\partial^2}{\partial t^2} + \kappa P(t) \right) a(z,t) \]

low-energy both solid-state and fiber oscillators

Reduced cubic-quintic complex nonlinear Ginzburg-Landau equation

\[ \frac{\partial a(z,t)}{\partial z} = i \left\{ \beta \frac{\partial^2}{\partial t^2} - \gamma P(t) \right\} a(z,t) + \left( -\sigma(E) + \alpha \frac{\partial^2}{\partial t^2} + \kappa [1 - \zeta P(t)] a(z,t) \right) \]

Kerr-lens mode-locked solid-state CPO and ANDi fiber oscillator mode-locked by polarization modulator

Cubic-quintic complex nonlinear Ginzburg-Landau equation

\[ \frac{\partial a(z,t)}{\partial z} = i \left\{ \beta \frac{\partial^2}{\partial t^2} - \gamma P(t) \right\} a(z,t) + \left( -\sigma(E) + \alpha \frac{\partial^2}{\partial t^2} + \kappa [1 - \zeta P(t)] a(z,t) \right) \]

an oscillator without strong mode confinement

Generalized complex nonlinear Ginzburg-Landau equation

\[ \frac{\partial a(z,t)}{\partial z} = i \left\{ \beta \frac{\partial^2}{\partial t^2} - \gamma P(t) \right\} a(z,t) + \left( -\sigma(E) + \alpha \frac{\partial^2}{\partial t^2} + \frac{\kappa P(t)}{1 + \zeta P(t)} \right) a(z,t) \]

semiconductor saturable absorber

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Method of approximate integration
(example of reduced cubic-quintic CNGLE)

\[ a(z,t) = \sqrt{P(t)} \exp[i\phi(t) - iqz] \]

the travelling wave reduction approximations:
1) \( T >> \sqrt{\beta} \)
2) \( \beta >> \alpha \)

\[ \phi \] is a rapidly varying function that allows applying the method of stationary phase to the Fourier image of field

Spectral profile and energy:

\[ p(\omega) \approx \frac{6\pi\gamma}{\zeta\kappa} \frac{H(\Delta^2 - \omega^2)}{\Xi^2 + \omega^2}, \]

\[ E = \frac{6\gamma}{\zeta\kappa\Xi} \arctanh\left(\frac{\Delta}{\Xi}\right) \]

Dimensionless energy replaces \( \Sigma \)

\[ \gamma P(t) = q - \beta \Omega(t)^2, \]

\[ \frac{d\Omega}{dt} = \frac{\beta\zeta\kappa}{3\gamma^2} (\Delta^2 - \Omega^2)(\Omega^2 + \Xi^2), \]

\[ \beta \Delta^2 = \gamma P_0 = \frac{3\gamma}{4\zeta} \left(1 - \frac{c}{2} \pm \sqrt{\left(1 - \frac{c}{2}\right)^2 - 4\Sigma}\right), \]

\[ \beta \Xi^2 = \frac{\gamma}{\zeta} (1 + c) - \frac{5}{3} \gamma P_0 \]

DUAL control parameters: \( c \) and \( \Sigma \)
Master diagram (structure).
Reduced cubic-quintic CNGLE

\[ b = \zeta \gamma / \chi = \infty \]

\[ c = \alpha \gamma / \beta k \]
\[ \Sigma = \sigma \zeta / k \]

Pulse is stable when \( \Sigma = 0 \)
Master diagram.

Cubic-quintic CNGLE (I)

\[ b = \zeta \gamma / \chi = \infty \]

\[ b = \zeta \gamma / \chi = 0.2 \]

\[ c \equiv \alpha \gamma / \beta \kappa \]
Master diagram.

Cubic-quintic CNGLE (II)

\[ b = \frac{\zeta \gamma}{\chi} = \infty \]

\[ b = \frac{\zeta \gamma}{\chi} = -5 \]

\[ c \equiv \frac{\alpha \gamma}{\beta \kappa} \]
Master diagram.

Cubic-quintic vs. generalized CNGLE

\[ c = \frac{\alpha}{\beta} \times \frac{\gamma}{\kappa} \approx 1 \]

\[ 0,1 \quad 1 \quad 10 \quad 100 \]

\[ 0 \quad 1 \quad 2 \]

\[ E \]

\[ c \]

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1A.C.Chong, W.H.Renninger, F.W.Wise, JOSA B25, 140(2008): \( E=12 \ \text{nJ}, \beta=0.1 \ \text{ps}^2, \alpha=450+2000 \ \text{fs}^2 \)

2S.Naumov et. al. NJP 7, 216 (2005): \( E=0.9 \ \mu \text{J}, \beta=220 \ \text{fs}^2, \alpha=2.5 \ \text{fs}^2 \)

thin-disk: \( E=30 \ \mu \text{J}, \beta=3000 \ \text{fs}^2, \alpha=900 \ \text{fs}^2 \)

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Spectral shapes.

Cubic-quintic CNGLE

positive branch \((c=1; \Sigma=0.01)\)

- \(b = -5\)
- \(b = 0.2\)
- \(b = 20\)
- \(b = -1.8\)

negative branch \((c=1; \Sigma=0.01)\)

- \(b = -0.2\)
- \(b = 0.2\)
- \(b = 20; -5\)

\[c = \alpha \gamma / \beta k\]
Spectral width.

Cubic-quintic CNGLE

\[ c = \frac{\alpha \gamma}{\beta \kappa} \]

\[ \Sigma = \frac{\sigma \zeta}{\kappa} \]
The developed analytical theory allows representing the CPO parametrical space (for both fiber and solid-state oscillator) in the form of two-dimensional master diagram.

As a result, the CSP characteristics become easily traceable.

From a uniform standpoint, the main difference between a fiber (ANDi) and a solid-state CPO is that they realize two different branches of CSP.

Such branches differ in the energy and spectral width scaling rules.

The spectrum profiles are found to be parabolic, flat-top, finger-like and concave.

The source of concave spectra is the quintic self-phase modulation.

The theory allows tracing and optimizing the CPO characteristics for different types of self-amplitude modulators and demonstrates the feasibility of energy-scalable CPO providing over 10 mJ output energy.

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