Phase Modulation of Radiation of Solid-State Lasers in the Presence of Kerr Optical Nonlinearity

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Abstract—The mechanism of ultrashort pulse (USP) formation in the presence of phase modulation and self-phase modulation is analyzed. The stability of USPs is studied.

INTRODUCTION

Kerr nonlinearity, which is responsible for self-phase modulation (SPM) of the laser field, was long thought to cause a decrease in mode-locking efficiency. This statement was based on the concept of an increase in USP loss through nonlinear expansion of the USP spectrum (see, for example, [1]). However, further studies showed the possibility of using SPM and self-focusing for the production of extremely short pulses. This feasibility is based on the transformation of the SPM-induced nonlinear phase of the laser field to the amplitude response, owing to the interference of the field with its replica or narrowing of the spatial cross section of a laser field on a diaphragm [2].

However, such devices possess a distinctive feature, the necessity of using the start mechanism based on a method of low-frequency modulation of laser resonator parameters (the introduction of rotating plates or prisms, "shaking up" of mirrors, etc.) [3]. This fact suggests the presence of an additional mode-locking mechanism for anomalously low frequencies of external modulation. This was demonstrated in [4], where the presence of this mechanism was attributed to SPM and the Doppler frequency shift by a moving mirror.

Moreover, the feasibility of producing extremely short USPs under conditions of phase modulation (PM) at a frequency matched to the reciprocal of the resonator period was demonstrated. Their duration is much smaller than the pulse duration provided by conventional PM, which is also associated with SPM of the laser field [5].

These facts give evidence of a new phase mechanism of mode locking, whose main regularities are still not clearly understood. Here, we analyze the mechanism of forming USPs of both symmetric and asymmetric shape in the presence of SPM and PM (including PM at a frequency that is not matched to the laser period). The stability of USPs is studied, and the change from the regime of conventional PM to mode locking due to SPM is demonstrated. The conditions for controlling phase and time characteristics of USPs in this regime are analyzed.

RESULTS AND DISCUSSION

The mechanism of USP formation in the case of PM is quite well studied (see, for example, [6]). Its principle consists in forming an additional phase shift (chirp) for field components offset from the point of stationary phase coincident with the maximum or the minimum of the phase modulation curve. As a result, the stronger the offset of field regions from the point of the stationary field, the stronger their "forcing out" from the gain band of the active medium (AM). Thus, we have amplitude modulation, and a USP is formed.

Parameters of a stationary USP, formed due to PM with frequency ω normalized to the depth of modulation, can be obtained from the equation [7]

\[ g + \alpha \frac{\partial^2}{\partial t^2} + i\phi \pm i\omega t^2 + \alpha_0 \alpha(t) = 0. \]  

Here, \( \alpha(t) \) is the laser field profile, \( t \) is local time, \( \alpha \) is the saturated gain of the AM, \( g = \alpha + i \) is the summary saturated gain, \( l \) are linear losses, \( \phi \) is the phase shift for a single round trip along a resonator, and \( \alpha_0 \) is the operator taking into account nonlinear effects associated with SPM. In equation (1), all the times are normalized to the reciprocal width of the spectral gain band \( t_p \) (pulse durations and frequencies are normalized in accordance with this rule as well).

In the absence of SPM (\( \alpha_0 = 0 \)), the pulse solution of equation (1) has the form \( \alpha(t) = \alpha_0 \exp(-t^2/t_p^2 + i\psi t) \). The amplitude is determined from the saturation conditions (see below), \( t_p = \frac{2\sqrt{g}}{\alpha_0^2} = \frac{\sqrt{8\alpha}}{\frac{1}{4} \alpha_0^2} \) is the pulse duration, \( \psi = \pm \frac{1}{2} \) is the chirp, \( \phi = \pm g \) is the phase retardation, and \( \alpha = \frac{g t_p^2}{2} \) is the saturated gain. The signs of chirp and phase retardation are determined from where
a USP is formed, at the maximum or the minimum of the phase modulation curve (both cases are completely equivalent).

In the presence of SPM (\(\hat{Q} = -i|q|^2\) [8]; for simplicity, field intensities are normalized to the SPM factor), the situation changes. In view of the fact that the additional SPM-induced chirp is positive, it is evident that it will cause an increase in chirp for the pulse formed at the minimum of the modulation curve and a decrease in chirp for the pulse formed at its maximum. In accordance with this statement, the USP duration will decrease (increase): 

\[
t_p = \frac{\sqrt{4g + 2a_0^2}}{|\omega|}, \quad \phi = -g + a_0^2.
\]

Thus, we obtain the conditions for additional narrowing of a laser pulse at the minimum of the modulation curve.

Now let us analyze the stability of USPs of this type. To study the stability, we assume that the amplitude perturbation of a USP is formed under conditions of chirping under the action of PM and SPM. Its exponential growth (or damping) from passage to passage will be determined by the increment \(\lambda\) representing an eigenvalue of the operator 

\[
Q + \alpha \frac{d^2}{dt^2} - (\omega^2 + a_0^2)^2,
\]

which is the operator of a quantum linear oscillator [9]. An USP is stable in the case where \(\lambda < 0\). This condition imposes limits on the sum of the saturated gain, which must be lower than the "excitation energy" of the ground energy level corresponding to \(\lambda = 0\) \((g < B^2/3C + 1/C/188, A = 4a_0^2, B = 2a_0^2, C = 9A + \sqrt{3}(27A^2 - 4B^2))\), or on the modulation frequency \((\omega > g/2\sqrt{1 + a_0^2/g})\).

Now we must take into account that the saturation of amplification determines the relation between the USP intensity and the saturated gain. It has the form [10]

\[
\alpha = a_\alpha \frac{1}{1 - \exp(-E)} \exp(-E),
\]

where \(E\) is the USP energy, \(P\) is the pump energy, and \(a_\alpha\) is the maximum gain for complete population inversion of the AM. For small pulse energies \((E \ll 1)\), the intensity is determined by the expression

\[
a_0 = \frac{g^2(1 - \zeta)}{4\pi a_\alpha^2 \omega^2 \left[ \frac{1 + 16\pi a_\alpha^2 \omega^2}{(1 - \zeta)^2 g} - 1 \right]},
\]

where \(\zeta = e^{2g}.\) When substituted to the conditions for the threshold modulation frequency, this expression gives the dependence of the minimum modulation frequency on the summary saturated gain. The solid curves in Fig. 1 illustrate this dependence for different pump energies. An increase in the pump energy causes a small decrease in the minimum modulation frequency (remember that the modulation frequency is normalized to the gain bandwidth and includes the depth of modulation). This dependence can be approximately expressed in the form

\[
\omega > \frac{g}{2} + \frac{3\pi g}{4 \sqrt{\alpha_\alpha/g\pi + 2\zeta - 2}},
\]

Note that the first term making the dominant contribution is determined by a simplified analysis of stability on the basis of studying three-dimensional mapping of parameters of the Gaussian pulse obtained from equation (1):

\[
\partial a_\alpha/\partial z = -2g\alpha, \quad \partial c/\partial z = -4(\alpha^2 - \psi^2)\alpha,
\]

\[
\partial \psi/\partial z = -6\alpha \psi + 2a_0^2 c,
\]

where \(z\) is the longitudinal coordinate and \(c = 1/t_p^2\).

It is evident that restrictions on USP stability under conditions of SPM determine its minimum duration, which is represented by curve 3 in Fig. 1. For comparison, curve 4 represents the USP duration in the case of conventional PM. One can see that the dependences of duration on the summary saturated gain are substantially different in these two cases. Starting from a certain \(g\), SPM enables one to produce shorter pulses (about 10–20\(t_p\)) than conventional PM. This is caused by the fact that an increase in \(g\) favors an increase in USP intensity and, therefore, additional pulse shortening due to SPM. In the case of conventional PM, an increase in \(g\) causes USP lengthening.

Of most interest is the analysis of the nature of destabilization of a USP formed at the minimum of the modulation curve in the presence of SPM; the USP perturbation spectrum is discrete in this case. This means that the pulse shape will be distorted. Eigenfunctions of
the equation describing the evolution of perturbations (see above) are expressed in terms of $H_n(it_{\text{p}})e^{\frac{b^2}{2}}$, where $H_n$ is the Hermitean polynomial. Their excitation may cause the loss of symmetry of the solution of equation (1) for the following reasons: First, the depletion of gain by a chirped pulse forms conditions for the shift of its carrier frequency to the red region [11]. Second, the behavior of the phase of these excitations is such that points of stationary phase, which are offset from the extremum of the modulation curve at $t = 0$, are formed. This is illustrated in Fig. 2a, where curves 1 and 2 show the behavior of phase for the first and second excited levels of the perturbation spectrum, respectively. One can see that, in the second case, points of stationary phase lying off the extremum of the modulation curve (points A) are formed and fluctuations may shift the pulse carrier from the point $t = 0$ (spontaneous symmetry breaking).

In the absence of SPM, USPs cannot be formed outside the extremum of the modulation curve, but SPM forms conditions for phase stabilization at other points of the modulation curve as well. This is illustrated by curve 1 in Fig. 2b, where the point of stationary phase A is formed due to the chirp produced by SPM. Thus, symmetry breaking in the system due to SPM will cause a pulse shift from the minimum of the modulation curve to the point of inflection near which PM may be described by the term $\text{iar}$.

In this case, as one can see from Fig. 2b (curve 1), the USP carrier frequency will have an offset with respect to the gain band center, and the presence of the point of stationary phase for $t \approx 0$ means that the leading (or the trailing, depending on the sign of $\omega$) edge of a pulse will be amplified more strongly, and this, in turn, will cause a USP shift in the resonator period. This shift can be described by the term $-\delta \frac{\partial}{\partial t}$ in equation (1), where $\delta$ is the USP retardation for the round trip of the resonator.

In this case, the process of USP formation is rather clear. In view of the fact that SPM causes the dispersion of spectral components about the central frequency, the USP spectrum in the presence of external frequency modulation will be tuned to the gain band center due to this nonlinear scatter of its low- or high-frequency region, depending on whether the frequency shift produced by external frequency modulation is anti-Stokes or Stokes. Under these conditions, a symmetric pulse may be formed only in the case where the central frequency has a shift $\Omega$ with respect to the gain band center. The shape of this pulse is approximately described by the formula

$$a(t) = a_0e^{\frac{b^2}{2}}.$$ 

The substitution of this pulse shape in equation (1) gives the expressions

$$t_{\text{p}} = \frac{2\sqrt{a_0^2(4g - 2a_0^3)}{\omega}}{[\omega]_0}$$

$$\delta = \frac{(2g - a_0^3)(\omega - \Omega a_0^3)}{a_0^3\Omega^3}$$

$$\Omega = \frac{\omega}{2a_0^3}, \quad \phi = -g + \alpha \Omega^2 + \Omega \delta + a_0^3$$

$$\alpha = \frac{2g - a_0^3}{2\Omega^2}, \quad \psi = \frac{1}{2}t_{\text{p}}$$

for USP parameters, gain, and time retardation on the resonator period.

The analysis of stability of USPs of this type by the method described above gives the following result. The system is characterized by the minimum modulation frequency determining the formation of a stable USP and increasing with increasing $g$ or $P$ (Fig. 3a, solid curves 1, 2). The existence of this frequency is associated with the necessity of "pushing out" those noise components for which SPM is weak from the gain band center. This determines the minimum USP duration (dashed curve in Fig. 3), which rapidly decreases with increasing $g$. However, in view of the fact that pulse stabilization in this case is governed by SPM, the system is characterized by the minimum intensity of stable USPs (Fig. 3b). This intensity increases with increasing modulation frequency, because pulse formation (i.e.,

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formation of the point of stationary phase) in this case requires a stronger chirp.

Of most interest is the fact that the minimum frequency of external modulation rapidly decreases with decreasing summary saturated gain, and its value under these conditions is lower by approximately three orders of magnitude than the value in the case of conventional PM. This means that mode locking can be achieved for anomalously low modulation frequencies and without matching to the resonator period. This regime was observed in [4]. In [3], it was used for initiating USP formation in a laser with an additional resonator. In both cases, frequency modulation was carried out due to mirror vibration (with a frequency of about 10 Hz; for the amplitudes of mirror motion used in the experiments, this gave a frequency shift in the range 10–100 kHz due to the Doppler effect), and the USP production was observed near the points of maximum mirror velocity (i.e., at the point of inflection of the modulation curve).

Taking into account higher orders in the expansion of the summary saturated gain in USP energy, the analysis suggests that the domain of existence for USPs of type (2) is bounded in the modulation frequency not only from below but from above as well. Note that the stability range rapidly narrows with increasing $g$ (Fig. 3a, curve 2'). The above statement is associated with the fact that, in the case under consideration, the USP intensity increases with increasing PM frequency, because the compensation of an external frequency shift requires stronger SPM, and this, in turn, decreases $\alpha$ [see equation (2)] and therefore decreases the excitation energy for levels of undamped perturbations. The
latter corresponds to an enhancement of PM contribution, not only to the suppression of the noise spectrum but to the perturbation of the USP spectrum as well. Thus, the presence of fluctuations at the initial stage of USP formation due to SPM may break phase and time symmetry, not only because of the frequency detuning of a pulse from the gain band center but because of the cubic dependence of the phase on the local time emerging under these conditions and the pulse shape distortion as well [5]. One can see from Fig. 2b that, in the case where the time dependence of phase has the form \( \omega t^2 + \omega t^3 \) (curve 2) or \( \omega t + \omega t^2 \) (curve 3), the system has points of stationary phase (A), which determines the possibility of pulse formation under these conditions.

In the general case, the pulse shape near the intensity maximum may be described in the form

\[
a = a_0 \exp \left[ b t^2 - \frac{t^4}{t_0^2} + i(\Omega t + \omega t^2 + \omega t^3) \right],
\]

where \( b \) is the parameter describing the time asymmetry of a USP.

Figure 4 illustrates dependences of the main parameters of a USP on its intensity (determined by the pump energy) for different time delays of a pulse on the resonator period. These delays can be varied by detuning the PM frequency with respect to the reciprocal of the resonator round trip.

One can see from Fig. 4 that, in the general case, a USP has both the frequency detuning from the gain band center and the time and phase asymmetry; also, its duration may reach 10 \( t_0 \). The introduction of additional pulse retardation on the resonator period increases the USP duration without increasing the minimum attainable duration but makes it possible to control phase and amplitude characteristics of a pulse (curves 2, 3). For example, one can eliminate the frequency detuning \( \Omega \) (points \( \alpha \)) or the shape asymmetry (points \( \beta \)) and the cubic phase (nonlinear chirp, point \( \gamma \)). The latter fact is of substantial importance, because it provides a means for efficient USP compression by way of counterbalancing chirp by the negative dispersion of group velocity.

CONCLUSIONS

In summary, nonlinear spectral processes under conditions of PM make it possible to control time and phase characteristics of USPs in wide ranges, decrease considerably their duration, increase the stability, and decrease strongly the frequency of external modulation. This result offers the greatest promise for femtosecond lasers, where phase characteristics of USPs substantially affect their stability, duration, and the feasibility of further amplification for the purpose of production of ultrastrong fields.

REFERENCES


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