Dark-Energy-Matter from Vacuum owing to the General Covariance Violation

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Abstract

It seems that the violation of covariance relatively the general coordinate transformations exists in the real world and contributes to its fundamental structure. This guess allows omitting the main part of the vacuum energy reasonably and considering a remaining part emergent from the zero-point fluctuations of the quantum fields as some “fluid” possessing energy density and pressure. Then, the equation of vacuum state and the speed of vacuum sound-waves result from the zero vacuum entropy assumption. Generally, a vacuum can be considered as a “basis” for dark-energy-matter unification. The resulting evolution of the background space-time metric resembles that of the Milne-like universe, but with the late accelerating stage.

**General covariance and quantum vacuum:** Clarification of the role of vacuum remains the key issue of cosmology despite the numerous hypotheses and suggestions [1]. One may highlight two different viewpoints. In the 1950s, the progress of the quantum field theory in the Minkowski space led to the idea of renormalization of the average value of an energy-momentum tensor for a quantum field on a curved classical background. That results in

\[ <0| T_{\mu\nu}|0> = A g_{\mu\nu} + B \left( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + ... F_{\mu\nu}(R), \]  

(1)

where \( F_{\mu\nu}(R) \) is some nonlinear function of the Riemann tensor. If one considers \( <0| T_{\mu\nu}|0> \) as a source of the gravitational field in the Einstein equations, the first term diverging as fourth degree of momentum renormalizes the cosmological constant, and the second quadratically diverging term renormalizes the Einstein gravitational constant [2]. The remaining terms \( F_{\mu\nu}(R) \) form a basis for the so-called \( F(R) \) theories. For dimension reasons, at least for the massless particles, a vacuum density is of the order of \( H^4 \) for an expanding Universe, where \( H \) is the Hubble constant. As the critical density \( \sim H^2 M_p^2 \), where \( M_p \) is the Planck mass, the vacuum turns out almost empty now, when the Hubble constant is small.

On the other hand, the conceptually simple structure of the Einstein equations postulating, that the energy-momentum tensor defines the space-time curvature, faces some difficulties in the explanation of astrophysical data. In particular, the rotational curves of spiral galaxies indicate the existence of a certain non-luminous (“dark”) matter, and the data demonstrating the accelerated Universe expansion hint at some “dark energy” acting like the Einstein cosmological constant.

According to the above approach, the dark energy and matter are some “substances” beyond a quantum manifestation of vacuum, at least for massless particles. An essential moment here is the general covariance of \( <0| T_{\mu\nu}|0> \). However, a fundamental problem is an evident impossibility to define a generally covariant vacuum state [2] except for some special classes of background metrics [3].

The question arises, whether \( <0| T_{\mu\nu}|0> \) should be covariant if an invariant vacuum state \( |0> \) is obscure? In this regard, one may assume violation of general covariance for gravity. Examples of this kind are Horava-Lifschitz theory [4], shape dynamics [5], unimodular gravity [6] and five-vector theory of gravity (FVT) [7], where a standard Einstein-Hilbert action is varied over some restricted class of metrics. Thus, new opportunities for identification of vacuum energy density with dark energy and dark matter emerge.

Here we consider FVT as a model for the fundamental mechanism of the general covariance violation and investigate a case with the ultra-violet (UV) cut-off of comoving momentums \( k \) connected with physical momentums by \( p = k/a \) (\( a \) is a scale factor). Since comoving momentums do not change with the Universe evolution, it seems consistent to cut off them by some constant at the Planck mass level [8, 9]. The first problem is to explain why the main part of
vacuum energy does not influence the Universe expansion. A possible answer is given by FVT claiming that the reference level of an energy density is defined up to some arbitrary constant on some restricted class of metrics.

Properties of “residual” vacuum energy are described as some “fluid” having an equation of a state (ES). It is “ether” which is absent for the Minkowski space-time owing to vacuum invariance relatively the Lorentz transformations, but not for gravity, where there is no a generally covariant vacuum state and, thereby, some preferred reference frame should exist.

**Vacuum as a fluid:** Let us consider a quantum massless scalar field $\hat{\phi}(\eta, x)$ on a classical background of uniform, flat, expanding Universe defined by metric:

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = a^2(\eta)\left(d\eta^2 - \tilde{\gamma}_{ij}dx^i dx^j\right),$$  \hspace{1cm} (2)

where $\tilde{\gamma}_{ij} = \text{diag}\{1, 1, 1\}$ is a Euclidean 3-metric.

The operators of the energy density and pressure are

$$\hat{\rho}_\phi = \frac{\hat{\phi}'^2}{2a^2} + \frac{(\nabla \hat{\phi})^2}{2a^2},$$

$$\hat{p}_\phi = \frac{\hat{\phi}'^2}{2a^2} - \frac{(\nabla \hat{\phi})^2}{6a^2}.$$  \hspace{1cm} (3)

Expansion over the plane-wave modes $\hat{\phi}(\eta, r) = \sum_k \hat{\phi}_k(\eta)e^{ikr}$ gives

$$\hat{\phi}_k(\eta) = \hat{a}_+^* k \chi_k^*(\eta) + \hat{a}_k \chi_k(\eta),$$  \hspace{1cm} (4)

where

$$\chi_k(\eta) = \exp\left(\frac{-i}{\sqrt{2a(\eta)}} \int_0^\eta \sqrt{k^2 - \frac{a''(\tau)}{a(\tau)}} d\tau\right) \frac{e^{i k r}}{\sqrt{2k^2 \sqrt{k^2 - \frac{a''(\eta)}{a(\eta)}}}}$$  \hspace{1cm} (5)

in the adiabatic approximation. The mean vacuum energy density of a scalar field is

$$\rho_v a^4 = \frac{a^2}{2V} \int_V \left< |\hat{\phi}'|^2 |0\right> + \left< 0 |(\nabla \hat{\phi})^2 |0\right> d^3r = \frac{a^2}{2} \sum_k <0|\hat{\phi}_k^* \hat{\phi}_{-k}|0 > + \frac{4\pi a^2}{2(2\pi)^3} \int_{k_{max}}^{k_{max}} \left(\chi_k^* \chi_k + k^2 \chi_k^* \chi_k\right) k^2 dk \approx \frac{1}{4\pi^2} \left(\frac{k_{max}^4}{4} + \frac{k_{max}^2 a^2}{4a^2} + O(a^3) + O(a'a') + O(a'' + ...)\right),$$  \hspace{1cm} (6)

where $V$ is some normalizing volume which equals unity for simplicity, $a^2$, $a'$ are 2nd-order of smallness, $a'^2$, $a''a'$ are 3d-order and so on. The first term diverging as $\sim k_{max}^4$ can be omitted in a frame of FVT. Calculation of the second term $\sim k_{max}^2$ gives the vacuum energy density

$$\rho_v = \frac{a'^2}{2a^6} M_p^2 S_0$$  \hspace{1cm} (7)
about of the critical density $M_p^2 H^2 / 2$ with using the UV cut-off at the Planck mass level $k_{max} \sim M_p$. Here

$$ S_0 = \frac{1}{2 M_p^2} \sum_k \frac{1}{k} = \frac{1}{M_p^2 (2\pi)^3} \int \frac{d^3 k}{2k} = \frac{k_{max}^2}{8 \pi^2 M_p^2}. $$

According to FVT, the entire concept of critical density loses its demarcating role between closed and opened Universes. Here, we consider a flat universe ad hoc. The following quantity is on the road to the vacuum ES:

$$ < 0 | \hat{\rho} \phi - 3 \hat{p} \phi | 0 > = - \frac{1}{a^2} \int \left( < 0 | \hat{\phi}^2 | 0 > - < 0 | (\nabla \hat{\phi})^2 | 0 > \right) d^3 r = - \frac{1}{a^2} \sum_k < 0 | \hat{\phi}_k^* \hat{\phi}_k^\prime | 0 > - k^2 < 0 | \hat{\phi}_k^\prime \hat{\phi}_k - k^2 \hat{\phi}_k^* \hat{\phi}_k \phi | 0 > = \frac{1}{a^2} \sum_k a (\chi_k^* \chi_k^\prime - k^2 \chi_k^* \chi_k) \approx \frac{1}{2 a^6} \left( a a'' - a' \right) \sum_k \frac{1}{k} + O(a^3) + O(a' a'') + O(a''') + ..., \quad (8) $$

which does not contain the terms $\sim k_{max}^4$. As a result,

$$ \rho_v - 3 p_v = \frac{1}{a^6} \left( a a'' - a' \right) M_p^2 S_0, \quad (9) $$

so that from Eqs. (7), (9)

$$ p_v = \frac{M_p^2 S_0}{a^6} \left( \frac{1}{2} a'' - \frac{1}{3} a^\prime a \right). \quad (10) $$

The master equation for the Universe evolution

$$ \rho_v' + 3 a^\prime (\rho_v + p_v) = 0 \quad (11) $$

is satisfied by Eqs. (7), (10) and allows considering a vacuum as some “fluid”. To find ES explicitly, the expression for $a(\eta)$ has to be found. When the Universe is extra-filled with a cold dust-like matter, the scale factor obeys

$$ - \frac{1}{2} M_p^2 a^2 + \rho_v a^4 + \frac{1}{2} M_p^2 \Omega_m H^2 a = const, \quad (12) $$

$$ M_p^2 a'' = (\rho_v - 3 p_v) a^3 + \frac{1}{2} M_p^2 \Omega_m H^2, \quad (13) $$

where $\Omega_m$ is a dimensionless density of matter and $H(\eta_0) = a'(\eta_0)$ is the Hubble constant at present time $\eta = \eta_0$, when $a(\eta_0) = 1$. Since Eq. (12) is satisfied up to some constant in the FVT framework, a main part of the vacuum energy can be “absorbed” by definition of $const = \frac{1}{2} H^2 M_p^2 (S_0 + \Omega_m - 1)$. Hence:

$$ H(a) = \frac{\dot{a}}{a} = \frac{1}{a^2} \frac{da}{d\eta} = \frac{H}{a^2} \sqrt{\frac{S_0 + \Omega_m - 1 - \Omega_m a}{S_0 a^2 - 1}}, \quad (14) $$
where a dot denotes the differentiation over the cosmic time $dt = adη$. Finally, $ES$ is

$$w_v = p_v/\rho_v = \left(1 - \frac{2}{3} \frac{a''a}{a'^2}\right) = \frac{2a^3\Omega_m - 3a^2(S_0 + \Omega_m - 1) + S(S_0 + \Omega_m - 1)}{3(a^2 - S_0)((a - 1)\Omega_m - S_0 + 1)},$$  

(15)

where we took into account $a' = a^2 H(a)$ and $a'' = a^2 H(a) \frac{d}{d a} (a^2 H(a))$.

The explicit calculation of the vacuum energy density leads to

$$\rho_v = \frac{H^2 M^2_p S_0 (S_0 + \Omega_m - 1 - a\Omega_m)}{2a^4 (S_0 - a^2)},$$  

(16)

The deceleration parameter $q(z) = -\frac{a'}{a} = \frac{\dot{a} + H}{H} - 1$ obtained from Eq. (14) is shown in Fig. 1. As a result of vacuum domination, the Universe behaves like the Milne one [11] for $z \geq 2$, where $q \approx 0$, but then an acceleration phase $q < 0$ develops.

![Figure 1: Deceleration parameter dependence on the redshift $z$. Solid black, gray solid and gray dashed curves correspond to the standard $\Lambda$CDM model, the vacuum domination model (14) of the present paper and the mean value of the observational data reconstruction [12], respectively. Thin dashed curves point the 1$\sigma$ and 2$\sigma$ error channels of the reconstruction [12].](image)

Eq. (11) allows defining the vacuum “sound-waves” speed. When a vacuum entropy is neglected, the sound speed becomes

$$c_s^2 = \frac{p_v'}{\rho_v'} = \frac{2(5a^5\Omega_m - a^3\Omega_m S_0 + (7a^2 S_0 - 9a^4 - 2S_0^2)(\Omega_m + S_0 - 1))}{3(a^2 - S_0)(5a^3\Omega_m - 3a\Omega_m S_0 + (4S_0 - 6a^2)(\Omega_m + S_0 - 1))}. \quad (17)$$

This points out that a vacuum could be perturbable, and represents not only dark energy but dark matter, as well.

**Milne-like Universe and reality:** The discovery of accelerated Universe expansion had been a big surprise. However, if the above view on a vacuum is true, one more surprise is expected. Namely, a stage preceding the acceleration
one is Milne-like, as it is shown in Fig. 2. Only the most radical Milne-like universe without acceleration stage was investigated to date \cite{13,14}, but it is still attractive owing to an absence of horizon problem and superfluity of inflation hypothesis. A longer nucleosynthesis epoch in these models insists on a high baryon density \cite{15,16} $\Omega_b \sim 0.2$ to produce an observable amount of helium. According to this and the previous section, “dark matter” consists not only from the baryonic matter. Deuterium should be produced later during the inhomogeneous history \cite{16} of the Universe, possibly in spallation processes. There is no problem to obtain Lithium cosmologically, as well. Also, the observed CMB spectrum peaks are reproduced qualitatively in the framework of the considered model \cite{17}.

**Masses and vacuum:** A massless quantum field is considered above. In the case of the massive fields, Pauli’s idea could be actual (see \cite{18} and Refs.). That is a contribution of masses to vacuum energy from bosons and fermions should compensate each other. As a result, the main part of vacuum energy density is

\[
\rho_v = \frac{1}{16\pi^2 a^4} \int_0^{k_{\text{max}}} k^4 \sqrt{k^2 + a^2 m^2} dk \approx \frac{1}{16\pi^2} \left( \frac{k_{\text{max}}^4}{a^4} + \frac{m^2 k_{\text{max}}^2}{a^2} + \frac{m^4}{8} \left( 1 + \ln \left( \frac{m^2 a^2}{4k_{\text{max}}^2} \right) \right) \right) \tag{18}
\]

Simultaneously, three different principles could explain why the main part of vacuum energy does not contribute to the Universe evolution. The term $k_{\text{max}}^4$ is omitted in the FVT gravity, where the energy reference level is arbitrary. The terms $\sim m^4$ are a pure mass contribution to the vacuum density. However, the condensates precipitate in the Standard Model of Electroweak Interactions to generate masses itself. A density of condensates has the same order of $m^4$. Overall compensation of $m^4$—terms including condensates should be considered. This problem stills unresolved yet, but it implies some unknown symmetry mass generating potentials in Lagrangian allowing the compensation with the accuracy at least of the order of $\sim m_\nu^4$, where $m_\nu$ is the neutrino mass. The only

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**Figure 2:** Dependencies of the vacuum ES (a) and the velocity of the scalar “sound waves” (b) on the Universe scale factor. Solid line - $S_0 = 2.3$, $\Omega_m = 0.3$, dashed line - $S_0 = 2.3$, $\Omega_m = 0.03$. 

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informative terms for particle physics are $\sim k_{\text{max}}^2 m^2$, which gives

$$m_H^2 + N_A m_A^2 + 6m_W + 3m_Z = 12m_t^2,$$

(19)

where the top quark mass is $m_t = 173.2$ GeV, the Higgs boson mass is $m_H = 125$ GeV, the charged vector boson mass is $m_W = 80.4$ GeV, the neutral vector boson mass is $m_Z = 91.2$ GeV, and $m_A$ is a mass of unknown $A$–bosons contributing with the $N_A$–weight. Thus, the physics of vacuum beyond the Milne-like stage of the Universe expansion anticipates unknown bosons: a single boson $m_A \sim 530$ Gev, or, for instance, four bosons $m_A \sim 265$ Gev.

**References**


