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SRS-Driven Evolution of Dissipative Solitons in Fiber Lasers
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12.1 Introduction

It is known that in optical media with anomalous dispersion, in particular in a fiber laser cavity [1–3], it is possible to generate spectrally limited stable pulses that are called optical solitons. It was also found [4–6] that the regime of normal dispersion is more suitable for generating high-energy femtosecond pulses. In this regime, the femtosecond laser oscillator generates chirped pulses, that is, pulses with frequency modulation across its spectrum. The energy of these pulses may be ten or even hundred times more than that obtained for conventional solitons generated in the laser cavity with anomalous dispersion.

A cavity with net normal dispersion can be designed in different ways, and the regimes of generated chirped pulses are classified accordingly. Haus in his paper [7] proposed to use a scheme with alternating cavity parts of negative and positive dispersion thus providing dispersion management (DM). If total dispersion of such a DM cavity is close to zero, the oscillator generates so-called stretched pulses characterized by strong variations of the pulse duration at its round-trip. If the net cavity dispersion is positive and large enough, the pulse varies slowly in the normal-dispersion (amplifying) part of the cavity taking the shape close to parabolic. Such pulses are called self-similar [8] or similaritons [9]. Strong spectral filtering in the cavity is necessary in this case [10] to obtain stable pulses. As shown in [11], the similariton and soliton regimes can be realized sequentially inside one cavity in the parts with different dispersion signs.

The concept of dissipative solitons provides a unique frame of understanding for the existence and properties (including nearly constant parameters) of chirped pulses that can form and propagate stably in all-normal dispersion laser cavities [12–14]. High energy of the dissipative soliton is reached by means of high pulse chirping and the corresponding increase in its duration at fixed peak power.

Introduced in the 1990s [15, 16] as an extension of the term soliton, the term dissipative soliton (DS) describes localized structures of an electromagnetic field in nonconservative systems, where energy exchange with environment becomes
important in addition to a balance between nonlinearity and dispersion (or diffraction). Therefore, a balance between gain and loss is also needed to form a stable structure (see [17, 18] for a review of the DS basic principles and their emergence in optics, hydrodynamics, biology, etc.). Since the gain-to-loss balance is a typical requirement for laser generation, the concept of DSs has been successfully applied for description of pulse dynamics in a laser cavity (see [19, 20] for a review), especially for a broad class of passively mode-locked lasers [21]. The DS concept is also useful for practical purposes implying for the new cavity designs, in particular, those providing generation of high-energy femtosecond pulses. Being intrinsically one-dimensional, stable, and hands-free, all-fiber design of femtosecond oscillators is one of the most attractive for these purposes.

Implementation of the DS concept in fiber lasers led to the generation of ~10 nJ pulses in a scheme with dispersion compensation [22] and 20 nJ pulses in an all-normal-dispersion (ANDi) fiber laser consisting of normal-dispersion elements only [23]. The pulses are externally compressed to <200 fs. It appeared that energy scaling is possible by the fiber cross-section enlargement. As a result, the pulse energy in fiber oscillators has been increased to ~100 nJ at ~100 fs duration [24].

In theory, stable chirped pulses in normal-dispersion cavity are usually described in the frame of cubic–quintic Ginzburg–Landau equation (CQGLE) applicable for small variations of the pulse parameters along the cavity; see, for example, [17, 19]. There exists an analytical solution of the equation [25] relevant to optical fibers in the case of relatively small chirp parameter $f$ defined as

$$f = \frac{T\Delta}{4}, \quad (12.1)$$

where $T$ is a pulse duration and $\Delta$ is its spectral width. Though the solution obtained in [25] is exact, it is applicable to the pulses close to spectrally limited ones, $f \approx 1$. In contrast, in the discussed experiments dealing with energy scaling in fiber lasers, net normal dispersion value is high and pulses with chirp parameter $f \approx 10–30$ are generated. So, new approaches are needed for solving a problem of limitations arising at the DS energy scaling by means of cavity lengthening.

In this chapter, we give an overview of our results on the theoretical description of highly chirped DS (HCDS) based on the approximate analytical solution of Ginzburg–Landau equation in high-chirp limit and its stability analysis [26, 27]. Following the developed analytical model describing stable HCDS solutions, we study experimentally the regime of highly chirped pulses in an Yb-doped fiber laser with passive mode-locking via nonlinear polarization evolution (NPE) and test the model predictions. The generated pulses are shown to have equal phase shifts induced by nonlinearity and dispersion, thus acquiring a linear chirp in accordance with the HCDS concept. The pulse chirp parameter grows almost linearly with the cavity length leading to nearly linear increase of the pulse energy. The linear scaling, in turn, is valid only up to some extent, beyond which the stability of the regime degrades significantly. The reason of the stability break is shown (both theoretically and experimentally) to be an excessive rotation of the polarization ellipse in the cavity. Therefore, polarization-maintaining fibers for increasing
the scaling range of the HCDS regime can be a good choice. As a result of this study, an all-fiber configuration free of NPE overdriving based on long PM and short SM parts was proposed [28]. In this configuration, it appears possible to increase the cavity length up to 120 m, thus generating stable DSs with energies $> 20 \, \text{nJ}$ and chirp parameters $f > 200$ being compressible down to $\sim 200 \, \text{fs}$ [29]. At this stage, a new limiting factor comes into play, namely stimulated Raman scattering (SRS).

The detailed analysis of the SRS-driven evolution of DS in such long cavities has been performed in the frame of nonlinear Schrödinger equation (NSE) in which SRS term is added, and the model has been compared with the experiments [30]. It appears possible to realize stable DS in the presence of strong SRS effect. Moreover, with a new laser scheme based on the critical elements described earlier and an additional Raman feedback loop, stable linearly chirped DSs at Stokes-shifted wavelength, so-called Raman dissipative solitons (RDS) were recently demonstrated [31]. Together with the main DS, the Raman DS (RDS) of different orders form a multicolor complex of coherent pulses with higher total energy that can be, under certain conditions, compressed to sub-100 fs duration. Potential applications of the RDSs and the DS–RDS complexes are discussed at the end of this chapter.

### 12.2.1 Modeling

Newell showed in 1974 [32] that, in approximation of slowly varying amplitude, the equation for an envelope of the electromagnetic wave has a universal form and may be reduced to the cubic Ginzburg–Landau equation (CGLE). This equation and its generalizations, for example, including different higher-order nonlinear and dissipative effects, are used as a basis for description of wave dynamics in various physical phenomena, from hydrodynamics and plasma to femtosecond light pulses [7]. It is also important that this class of equations has soliton solutions in the broad range of physical parameters. It is clear that the solitons are observable in the experiment if they are stable. Therefore, many papers are devoted to the numerical studies of CGLE solutions stability, see, for example, [17] and citations therein.

As mentioned in Section 12.1, a physically important class of the soliton solutions of Ginzburg–Landau equation is the highly chirped dissipative solitons (HCDS). Strongly inhomogeneous phase distribution characterizes physically their capacity to redistribute energy without loss of stability. As will be demonstrated later, this property can provide energy scalability of HCDS. In the case of slow saturable absorber, whose relaxation time is comparable to or greater than the pulse duration, the CGLE can be solved exactly [33]. The solutions obtained
in that paper have a spectral profile with sharp edges, typical for DSs. In the case of fast nonlinearity, where the response time of the medium is much shorter than the pulse duration, attempts of the DS description in the frame of CGLE approach revealed that the solutions are unstable against amplitude blow up [34]. A reason of such behavior is physically obvious for positive self-amplitude modulation (SAM): an intensity growth enhances a nonlinear gain that induces a further increase of intensity. As a result of such positive feedback, the peak power increases unlimitedly. Physical senselessness of such a solution is obvious: a nonlinear gain cannot grow unlimitedly. Hence, one may conclude that the CGLE should be modified by an inclusion of SAM saturation.

From this point of view, a simplest master equation for the envelope \( E(z, t) \) may be rewritten as [17]

\[
\frac{\partial A}{\partial z} = iL \left( \frac{\beta_2}{2} \frac{\partial^2}{\partial t^2} - \gamma |A|^2 \right) A + \left( -\sigma + \alpha \frac{\partial^2}{\partial t^2} + \kappa |A|^2 (1 - \zeta |A|^2) \right) A, \tag{12.2}
\]

which is usually named as the CQGLE. Here, \( z \) coordinate is normalized by cavity length \( L \). The first two terms describe group-delay dispersion (GDD) and self-phase modulation (SPM) distributed along the cavity; therefore, they grow with \( L \). In long cavities (in the HCDS regime), these terms are much higher than the remaining four-point action terms describing a difference between loss and gain (\( \sigma \)), spectral filtering (\( \alpha \)), SAM (\( \kappa \)), and SAM saturation (\( \zeta \)) at a cavity round-trip. All the parameters are positive for the chosen designation. Spectral filtering is naturally present due to a finite width of the gain spectrum and may be enhanced by insertion of the additional filters into the cavity. SAM is implemented in experiments in various ways, namely by Kerr lensing [6], nonlinear optical loop mirror (NOLM) [4, 35], saturable absorber [36], or via nonlinear polarization evolution (NPE) [5, 7], thus providing mode-locking.

Equation (12.2) is applicable if variations of the pulse parameters are small at the cavity round-trip. As was shown in [37], Eq. (12.2) is applicable for the description of high-energy HCDSs formation in a Ti : Sa laser. The CQGLE also proved its effectiveness in the modeling of fiber lasers [14, 38–40]. A typical block scheme of ring fiber laser, which can be considered in the framework of CQGLE, is shown in Figure 12.1a.

The complex pulse envelope \( A(t) \) averaged along a cavity can be found as a solution of Eq. (12.2) under the stationarity condition \( \frac{\partial A}{\partial z} = iqA \), where \( q \) is the wave vector shift. An exact weakly chirped dissipative soliton solution of the CQGLE, which exists in a region of normal GDDs has been first demonstrated in [41]. Such a soliton can be expressed in the form \( A = a(t)^{1+f''} (f'' \text{ corresponds to dimensionless chirp parameter introduced earlier in the limit } f'' \sim f \gg 1) \) and its existence is provided by some fixed algebraic relations between the parameters of Eq. (12.2) [25]. Stability analysis and study of existence domain for this chirped soliton solution were performed in [25, 42, 43]. The high chirp limit \( f \to \infty \) requires that changes of a field phase prevail over the field amplitude that corresponds to the following conditions imposed on the system parameters: spectral filtering is much weaker than the round-trip GDD (\( \alpha \ll \beta_2 L \)), and SAM
is much weaker than the net SPM ($\kappa \ll \gamma L$). These conditions have to be fulfilled simultaneously to enhance the phase effects causing pulse chirping. In this limit, the chirp parameter is expressed simply via coefficients describing equation parameters (see [25], for instance):

$$f \approx \frac{3L}{(2\alpha/\beta_2 + \kappa/\gamma)}.$$  

(12.3)

The exact solution [25, 41] becomes singular for $f > \sqrt{3}$; that is why powerful technique to analysis of HCDSs with $f \gg 1$ based on the WKB ideology was invented in [26, 44]. Such an approach has been expanded in various generalizations of the nonlinear Ginzburg–Landau equation [45, 46]. In the next section, we present a sketch of the theory for energy scalable HCDSs.

### 12.2.1.1 Analytical Solution of CQGLE in the High Chirp Limit

In a nutshell, an HCDS solution of Eq. (12.2) in the limits of $f \gg 1$; $\alpha \approx \beta_2 L/2f \ll \beta_2 L$ and $\kappa \approx \gamma L/f \ll \gamma L$ can be expressed in the following way [26, 27]. A scaling in the time domain $t = tf$ allows expressing a wave envelope in the form

$$A(z, t) = a(\tau, z)e^{i\Psi(\tau, z) - iqz},$$  

(12.4)

where $\phi(t) = f\Psi(t/f, z)$ is a phase. The physical sense of such a scaling is that a chirped pulse is stretched by $f$ times in the time domain compared with a transform-limited pulse. In this case, the time ($\tau$) derivatives of $a$ and $\Psi$ become finite in the limit of $f \to \infty$, and we can define explicitly the vanishing terms of Eq. (12.2). Substituting Eq. (12.4) in Eq. (12.2) and separating its real and imaginary parts results in two equations for $a\Psi$ and $a\phi$ (see [27]).

The first principal step consists of the assumption that for an HCDS ($f \gg 1$) one may neglect the terms of the order of $O(1/f^2)$. In a steady-state regime with $(\frac{\delta \Psi}{\delta z} = 0)$, it allows obtaining the strict relationship between an instantaneous power $P(t) = P(\tau f) = a^2(\tau)$ and an instantaneous frequency $\Omega(t) = \frac{\delta \phi}{\delta t} = \frac{\delta \Psi(\tau)}{\delta \tau}$:

$$P(\tau f) = P_m - \frac{\beta_2}{2\gamma} \Omega^2(\tau f),$$  

(12.5)
where \( P_m = q/\gamma \) is a peak power at a pulse center where \( \Omega \equiv 0 \). Thus, the shape of a pulse is defined explicitly by its instantaneous frequency. The requirement \( P(\tau f) > 0 \) in Eq. (12.5) leads to the conclusion that the spectrum has to be truncated at some maximum frequency deviation \( \Delta \):

\[
\Delta^2 = \frac{2\gamma P_m}{\beta_2}. \tag{12.6}
\]

It is important that the theory allows explicit expressing an instantaneous frequency in the form of first-order ordinary differential equation (for details see [27]).

The second principal step concerns excluding of the physically meaningless solutions corresponding to \( \frac{d\Omega}{dt} = \infty \), that can be realized by the regularization technique \([26, 47]\) removing the divergences. It leads to the two-valued expressions for the HCDS peak power \( P_m \):

\[
P_m^\pm = \frac{3}{8\xi} \left( 2 - C \pm \sqrt{(2 - C)^2 - 16 \frac{\sigma_\xi}{\kappa}} \right), \tag{12.7}
\]

where parameter \( C \equiv 2\alpha_\gamma / \kappa \beta_2 \) is introduced. Thus, there exist only two nonsingular HCDSs of CQGLE; its instantaneous frequency can be expressed in the following form:

\[
\frac{d\Omega}{dt} = \frac{\kappa \zeta}{3\gamma L} \left[ \Omega^2(t) + \frac{2\gamma}{\beta_2 \zeta} (1 + C) - \frac{10\gamma}{3\beta_2} P_m \right] \left( P_m - \frac{\beta_2}{2\gamma} \Omega^2(t) \right). \tag{12.8}
\]

The solution to this equation may be written in an implicit form \([26]\) for both positive \((P_m = P_m^+ )\) and negative \((P_m = P_m^-)\) branches of Eq. (12.7):

\[
\text{arctanh} \left( \frac{\Omega(t)}{\Delta} \right) + \frac{1}{R} \text{arctan} \left( \frac{\Omega(t)}{R\Delta} \right) = t / T. \tag{12.9}
\]

Here, \( \Delta = \sqrt{\frac{2\gamma}{\beta_2} P_m} \) defines the width of a truncated HCDS spectrum, \( T = \frac{6\gamma^2 L}{\beta_2 \kappa \Delta (1 + R^2)} \) is a characteristic time that defines a pulse duration, and \( R \) is a parameter defining the pulse shape. The last can be expressed as

\[
R = \sqrt{\frac{1 + C}{\zeta P_m} - \frac{5}{3}}. \tag{12.10}
\]

The third principal step of the theory is its representation in the Fourier domain. With a scaled time variable \( t = f \tau \), one may come to

\[
A(\omega) = f \int d\tau a(\tau) e^{i f (\Psi(\tau) - \omega \tau)}. \tag{12.11}
\]

In the high-chirp limit of \( f \to \infty \), the integral can be calculated by the method of stationary phase that results in the following asymptotical expression \([48]\):

\[
A(\omega) = f a(\tau^*) e^{i f (\Psi(\tau^*) - \omega \tau^*)} \cdot \sqrt{\frac{2\pi}{-i f \frac{d^2 \Psi}{dt^2} |_{\tau = \tau^*}}} (1 + O(1/f)), \tag{12.12}
\]
where $\tau^*(\omega)$ is the point of stationary phase defined by condition $\frac{d^2\Omega}{d\tau^2} = \omega$ for any frequency $\omega$. Taking into account of $a^2(\tau) = P(t) = P_m - \frac{\beta_1}{2\tau} \Omega^2(t)$ and $\frac{d^2\Omega}{d\tau^2} = f \frac{d\Omega}{d\tau}$ that is defined by Eq. (12.8) result in [27]

$$I(\omega) \equiv |A(\omega)|^2 \approx \frac{6\pi\gamma L}{\zeta \kappa} \frac{1}{\omega^2 + R^2 \Delta^2} (1 + O(1/f)) \text{ at } |\omega| < \Delta.$$  \hspace{1cm} (12.13)

At $|\omega| > \Delta$, there is no point of stationary phase and, thus, the integral (Eq. (12.11)) tends to zero ($I(\omega) = 0$) asymptotically for $f \to \infty$.

As a result, the theory allows expressing the HCDS energy:

$$E \equiv \int \frac{d\omega}{2\pi} I(\omega) = \frac{12\pi\gamma L}{\zeta \kappa \Delta} \frac{\text{arcctg}(R)}{R}. \hspace{1cm} (12.14)$$

The most impressive phenomenon, which is revealed by the theory presented, is an energy scalability of HCDSs. As one can see, the HCDS shape $P(t)/P^\pm = B(t/T)$, spectral profile $I(\omega)/I(0) = A(\omega/\Delta)$, and pulse energy $E$ depend on the only parameter $R$. If all six parameters of the initial equation are changed at an $R$ value kept fixed, then the HCDS shape in time domain and its spectrum will change in a self-similar way, that is, its amplitude, duration, and chirp as well as the spectral amplitude and width will scale with the conservation of soliton shape. In the limiting case of $\xi P_m \ll 1 \ (R \to \infty)$, the solution $P(t)$ tends to $\text{sech}^2(t/T)$, that is, to the transform-limited CGLE solution. However, an opposite limit ($R \to 0$) corresponds to an HCDSs with rectangular profile.

Pulse energy, see Eq. (12.14), in these limiting cases tends to $\lim_{R \to 0} E = 0$ and $\lim_{R \to \infty} E = \infty$. The last limit for rectangular pulse reveals the phenomenon of perfect energy scalability of HCDSs, which is analogous to the dissipative soliton resonance reported in [49–51], and can be explicated in the form of so-called “master diagram” [44]. An iso-gain curve $\sigma = 0$ on the master diagram corresponds to condition of the HCDS marginal stability and the dissipative resonance condition for it is $\lim_{C \to 2/3} E \kappa^{3/2} \sigma^{1/2}/\gamma \alpha^{1/2} = \infty$. If the parameters of spectral filtering, SAM, and net-gain $\sigma$ are kept on some constant level, the energy scaling can be provided by simple scaling of fiber length $L$.

It is important to note, that HCDS solutions exist in the limited regions of the whole domain of CQGLE parameters [27]. Since the pulse power must take only positive real values, it follows from Eq. (12.7) that the HCDS solutions are absent in the region $C \equiv 2\sigma\gamma/\beta_2 \kappa < 2 - 4 \sqrt{\frac{\sigma^2}{\kappa}}$. The definitional domain for analytical solutions is shown in Figure 12.2 on the $(C, \frac{\sigma^2}{\kappa})$ plane. Numbers (I) and (II) denote the regions where HCDSs do not exist [27]. The condition of the pulse spectrum limitation $R^2 > 0$ defines the HCDS existence regions for the positive ($P_m = P_m^*$) (IV) and the negative ($P_m = P_m^-$) (III, IV) branches. In this diagram, the dissipative resonance condition $(R = 0)$ corresponds to the dividing border between the regions III and IV.
12.2.1.2 Comparison of Analytics with Numerics

It is important to analyze the stability of both branches of HCDS existing in the regions III and IV. Such an analysis was based on the numerical solution of Eq. (12.2). The technical details of the calculations can be found in [27].

In region IV of Figure 12.2, the points with coordinates \((C, \sigma \zeta / \kappa)\) equal to \((0.9, 0.04), (0.5, 0.13), (0.75, 0.006), (1.5, 0.007)\) are marked by the numbers 1, 2, 3, and 4, respectively. For each point, we performed several simulations with various initial conditions including a Gaussian shape pulse, an analytical solution, and white noise [27]. It was found that for any parameter set inside region IV, there exists a stable analytical solution (positive branch). Numerical simulation with any initial perturbation converges to this solution. On the contrary, in region III, it was not possible to obtain any stable solution, even if the analytical solutions for negative branch were taken as the initial approximation. All the initial approximations either decay after 300–500 round-trips or broaden unlimitedly in time domain at the constant amplitude. Note that a similar behavior has been observed in [42] for the negative branch of the solution for weakly chirped soliton [41].

The obtained results lead to the conclusion that the negative branch solution is unstable in its all-definitional domain (regions III and IV), whereas the positive branch solution is stable in its region of existence (IV).

In region II, both solution are absent since \(R^2 < 0\). We performed simulations for several points in regions I and III with different parameters of the initial equation. It was found that in region I there exist stable solutions corresponding to weak chirp \(f \approx 1\), whereas highly chirped solutions \(f \gg 1\) are absent in this region. At the same time, we have not succeeded in realizing any stable either weakly or highly chirped solution in region III. Therefore, it is possible to
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![Figure 12.3](image)

**Figure 12.3** (a) Comparison between numerics (color) and positive branch of the analytical solution (solid black) for the spectral shape. (b) Time-domain shape of the positive branch (solid) and fitting (dashed) by parabola and sech$^2$. Additional black curve shows analytics in the limit $R \rightarrow 0$ ($R = 0.1$).

conclude that region IV is the only domain of existence and stability for all the highly chirped solutions of Eq. (12.2).

We also compared numerical simulation and analytical solution in points 1–4 of region IV in Figure 12.2. For all these points, the result of numerical simulation coincides with the analytical solution for positive branch in all its definitional domain (Figure 12.3). This means that the shape of the HCDS solution of the Eq. (12.2) at high chirp condition actually depends only on the value of parameter $R$ being independent of other parameters of the equation. The values of parameter $R$ for curves intersecting the points 1, 2, 3, and 4 shown in Figure 12.2 are equal to $R \approx 1.03, 0.65, 0.48, 2.45$, correspondingly. The performed numerical analysis has shown that along these curves the solutions are self-similar, that is, the shape of the pulse and the shape of its spectrum do not change.

In Figure 12.3, the spectral and temporal shapes of HCDSs are shown at various values of parameter $R$, corresponding to dashed lines in Figure 12.2. Analytical solutions are obtained from Eqs. (12.5) and (12.9). According to discussion in the analytical section, at $R = 2.45$ the pulse shape is approximated well by the expression $\text{sech}^2$, that is, is close to the shape of standard NLSE soliton. With decreasing $R$ the shape is changed and tends to parabolic one in point 3 (near the left border of region IV). In the limiting case of $R \rightarrow 0$ (exactly at the left border of region IV), the pulse takes the shape close to rectangular one: $P = P_m^+(1 - R^2 \tan^2(R t/T))$ at $t < \frac{\pi}{R}.$

Since exact solutions of CQGLE [25, 41–43] are singular at high chirp, our demonstration of stable HCDSs of CQGLE has high theoretical and practical significance [27, 44]. Only one of two solutions appears to be stable (+ branch in notation of paper [26]) in the absence of gain saturation. This is similar to the case of weakly chirped solitons [25, 41–43]. This fact is rather common and follows from more general mathematical theory of bifurcations. In addition, direct comparison of the analytical solutions for the pulse spectra with the numerical ones demonstrates the $10^{-2} - 10^{-4}$ accuracy for chirp parameters $f > 10$. The existence domain for the stable HCDS solutions with possibility of its self-start has been found in the CQGLE parameters set. Meanwhile it has been shown that the stable solutions correspond to the HCDS family with only one parameter.
R described by the expression (12.10). Inside this family the pulse shape is changed from the conventional soliton shape of \( \text{sech}^2 \) at \( R \to \infty \), to rectangular one at \( R \to 0 \), with a shape close to parabolic one in between: at \( R \sim 0.5 \). This parameter may be calculated for any experimental configuration and one can perform a quantitative comparison with experiment that makes a significant step in development of analytical theory.

The obtained theoretical results make it possible to classify experimentally observed HCDS pulses and to optimize experimental schemes aiming at the generation of pulses with various shapes.

12.2.2

**Experiment and its Comparison with Simulation**

Since the approximate analytical solution is applicable in the high-chirp limit, \( f \gg 1 \), it corresponds to conditions of most experiments with fiber lasers of ANDi type generating highly chirped pulses (HCP) [23, 29, 52, 53]. This model also shows that spectral shaping may be achieved even without strong spectral filtering produced by an additional filter element. Stability analysis of the solution [26] performed in papers [27, 54] has shown that only one of two solution branches appears to be stable. At that, stable solutions may be represented as one-parameter family with composite parameter \( R \) defining the pulse shape, as shown in Section 12.2.1.

In this section, we explore the developed analytical model [26, 27] for verification of the HCDS regime in the experiment, for optimization of the experimental scheme and for testing of energy scaling possibilities by means of cavity lengthening.

A scheme of the experimental setup that we use to study the regimes with HCP in a Yb-doped fiber laser [28] is presented in Figure 12.1b. The scheme is similar to those in [13, 23], but with one significant difference—we do not use any additional spectral filters for achieving the HCP regime. The laser consists of fiber and bulk optics parts. The fiber part includes (clockwise starting from the left top side): WDM coupler with 45 cm and 30 cm fiber tails, 15 cm active single-mode fiber (SMF) with high Yb\(^{3+}\) concentration, 80 cm passive XP1060 SMF of variable length and isolator with 30 and 85 cm tails. In the bulk optics part, we have quarter-wave (QWP) and half-wave plates (HWP), polarization beam splitter (PBS), 4% plate to monitor intracavity parameters of the laser and second QWP to produce slightly elliptic polarization at the input of the fiber part, necessary for NPE effect.

As the active fiber is highly doped its absorption amounts to 900 dB m\(^{-1}\) at 976 nm. A polarization insensitive fiber isolator has 2 dB losses at 1030 nm. The pumping is produced by a single-mode laser diode (LD) of up to 400 mW optical power at 976 nm. A monitoring port formed by 4% plate is used for measurements of the intracavity power and spectra. An output pulse train is observed on the output port of PBS used for output pulse width and power measurements. The pulse width is measured by scanning interferometric autocorrelator operating in
the range from 10 fs to 6 ps. The pulse spectrum is measured by Yokogawa optical spectrum analyzer with wavelength range from 600 to 1700 nm and wavelength resolution of 0.02 nm. The pulse train is also monitored by a fast oscilloscope in combination with a 1-GHz photodiode.

In the experiment, we have realized the HCP regime at four different values of cavity length: 4.5, 3.6, 3.2, and 2.6 m with the corresponding repetition rate 44, 55, 63, and 78 MHz. In all configurations, the studied HCP regime is found to be stable. Its self-starting is achieved by means of HWP tuning, without any external amplitude modulator. The threshold pump power for stable mode-locking is varied from 250 to 350 mW. At lower powers, the laser generates CW radiation or stochastic pulses. At higher power, the mode-locking is unstable and tends to CW regime. Note that the power range of stable operation is significantly reduced in case of the longest cavity.

Several measurements of spectrum, autocorrelation trace and laser power have been taken for each cavity length. Both the intracavity and extracavity spectra have steep edges with more than 30 dB contrast that is typical for HCDS regime. Full width of the intracavity spectrum at −10 dB level is in the range of 12−14 nm. The measured output pulse width varies from 1.3 ps for the shortest cavity to 3 ps for the longest cavity. The pulses were compressed externally by diffraction grating pair to about 250 fs [28].

The energy of output pulses, measured at the output port of PBS is presented in Figure 12.4a. Dotted line is a linear fit for the first three groups of points. Different points in one group correspond to mode-locking regimes with different level of pump power and slightly different adjustments of the phase plates, correspondingly. The energy value for the fourth group of points deviates significantly from the linear dependence and, therefore, this group was excluded from the fitting. We consider this point and reasons of its deviation in later sections. Note that the intracavity power is ~2 times higher than the output one and amounts to 1 nJ.

We can also calculate chirp parameter (Eq. (12.1)) using measured values of the full spectral width Δ and duration of the chirped pulse T. The dependence of

![Figure 12.4](image.png) (a) Energy of the output pulses depends on cavity length; inset – dimensionless chirp parameter defined by Eq. (12.1); (b) nonlinearity and dispersion phase shift ratio over one round-trip in the cavity [28].
the dimensionless chirp parameter versus cavity length is presented in the inset in Figure 12.4a. Note that the chirp parameter value at the longest length is also out of the linear trend, similar to the pulse energy. Let us compare the obtained experimental results with theoretical relations [26, 27].

Since our experimental results are in the area of applicability of the analytical solution, we should observe the same behavior in the theory and experiment. One of the important consequences of the theory is possibility of scaling provided by the linear dependence of chirp parameter on the cavity length (see Eq. (12.3)). As can be seen from the experiment (inset in Figure 12.4a), it is true for three experimental points with cavity lengths shorter than 4 m, but at the fourth point the chirp parameter deviates significantly from the linear dependence, similar to the pulse energy (Figure 12.4a). Reasons of this deviation will be also discussed later.

Another important theoretical issue characterizing HCDS is a balance between nonlinearity and dispersion phase shifts (Eq. (12.6)), so in theory their ratio should be equal to unity (as presented in Figure 12.4b). The figure also presents experimental points for this ratio calculated from the measured data and the specified fiber parameters. Average peak power \( P_0 \) defining nonlinear phase shift is calculated as \( P_0 = \sum_{i=1}^{3} P_i L_i / \sum_{i=1}^{3} L_i \), where \( P_i \) and \( L_i \) is the peak power and length in the \( i \)th section of cavity correspondingly (see Figure 12.1b). We have divided the cavity into three sections having different power levels. First one is between the fiber input and active fiber (minimum peak power). Second one is between active fiber and isolator (maximum peak power). Third one is between isolator and PBS.

The derived experimental points are marked by different colors/signs corresponding to different lengths (as in Figure 12.4a). The points locate around theoretical value in the region between 0.8 and 1.2 for all pump power and length values. We can, therefore, conclude that highly chirped pulses obtained in the experiment can be attributed to dissipative solitons. It is an important result meaning that a femtosecond oscillator can be described analytically by the developed theory in its applicability domain, \( f > 10 \). In order to explain significant deviation of the last point in length dependence (corresponding to the longest cavity length) both for energy and chirp parameters, we have applied numerical simulation described in the next section.

12.2.3 NPE Overdriving and its Influence on Dissipative Solitons

As follows from the experimental data (Figure 12.4) simultaneous linear growth of the chirp parameter and pulse energy with the cavity length breaks at some critical point (corresponding to cavity length of 4–4.5 m) that contradicts the analytical model predicting unlimited linear growth. To identify reasons leading to such critical behavior, we should go beyond the scalar model of CQGLE and perform numerical simulation in vectorial NPE model.
To describe directly polarization vector evolution, we should treat vectorial equation for electromagnetic field amplitude $E(z, t)$ (see, e.g., [55]):

$$\frac{\partial E_{\pm}}{\partial z} = \left[ \frac{i \beta_2}{2} \frac{\partial^2}{\partial t^2} - i \gamma I \pm \frac{i \gamma}{3} \left( |E_+|^2 - |E_-|^2 \right) \right] E_{\pm}$$

$$+ \frac{g_0}{1 + \frac{w_s}{W_s} \left[ 1 + \frac{1}{\Delta_1^2} \frac{\partial^2}{\partial t^2} \right]} E_{\pm}, \quad (12.15)$$

$$I = |E_+|^2 + |E_-|^2, \quad W = \int I(t) \, dt = \int \bar{I}(\Omega) \frac{d\Omega}{2\pi}. \quad (12.16)$$

where $W_s = P_s T_R$ is the saturation energy, $T_R = L n_0 / c$ is the cavity round-trip time, $\Delta_1$ is the half width of the gain spectrum, $g_0$ is the small-signal gain coefficient, and $E_{\pm} = (E_1 \pm i E_2) / \sqrt{2}$ are right-hand and left-hand circular polarizations of the electric field. In the simulation, we use experimental values of the dispersion, nonlinearity, and loss coefficients presented in Section 12.2.2. At each cavity length, an optimization of the output power is performed by means of the waveplates rotation thus changing polarization state. The calculated output pulse energy grows linearly with the cavity length. The growth is terminated at some critical length similar to that in experiments [28]. The higher the polarization ellipticity the lower the critical length and maximum pulse energy, accordingly. Herewith, the value of polarization rotation angle tends to $\pi / 2$ at the critical length thus leading to ambiguity in SAM.

These results agree in general with previous results on normal-dispersion cavity fiber lasers [13, 23, 56] where NPE “overdriving” was mentioned, but here we clarify in detail the physical mechanism of limitations at cavity lengthening, explained on the base of the vectorial NPE model [55]. Note that the existence of critical length was mentioned in paper [56] where similar pulse energy level ($\approx 1 \text{ nJ}$) has been obtained in 18-m long (9.9 MHz) cavity of Er-doped fiber laser generating linearly chirped pulses compressed down to $\approx 200 \text{ fs}$. In paper [57], a possibility of energy scaling by cavity lengthening of Yb-doped fiber lasers is analyzed on the basis of numerical simulation demonstrating gradual degradation of stability at lengthening from $\approx 10$ to $\approx 100 \text{ m}$.

The presence of critical length and its value in our experiments agree with the performed numerical simulation based on vectorial model for NPE effect [28]. The relatively low value of critical length in our case is defined by significant losses in bulk optics part of the cavity and light input–output of the fiber. At that, the reason for the HCDS regime break at critical length is found to be defined by the excessive ($> \pi / 2$) rotation angle via NPE effect in the fiber leading to instabilities/stochasticity and multiple pulse regimes. For lower losses and lower ellipticity of the pulse polarization at the fiber input, the model predicts larger critical length, but its maximum value will be limited by random birefringence of the fiber and its sensitivity to environment.

Based on the earlier results, we can propose a possible way how to overcome this limit. To keep the rotation angle lower than $\pi / 2$, one should increase the total length of the cavity by means of the polarization maintaining (PM) fiber in which
rotation is absent, whereas the length of the conventional non-PM SMF fiber part should be fixed (or even shortened) to keep the NPE-induced rotation angle within $\pi/2$ limit. As a result, nonlinear phase shift (and corresponding dispersion shift) accumulated along the PM-fiber part will lead to the HCDS pulse formation, while nonlinear polarization rotation in SMF part will provide stable mode-locking. The experiments in this configuration confirm the absence of NPE overdriving effect, but revealed new limiting factor of cavity lengthening, namely SRS resulting in DS energy conversion into noisy Raman pulse (RP) generated at the Stokes-shifted wavelength. This effect will be considered in detail in the next section.

12.3 Scaling of Dissipative Solitons in All-Fiber Configuration

12.3.1 Different Ways to Increase Pulse Energy, Limiting Factors

Scaling of the pulse energy in the ANDi laser generating highly chirped DSs is possible by increasing its cavity length or mode-field diameter of the fiber. Outstanding results on energy scaling were obtained by implementing intracavity large-mode-area (LMA) photonic crystal fibers [24, 36]. Peak power of generated pulses achieves level of about 1 MW, which is close to the self-focusing threshold in the fiber of $\sim 2$ MW [58]. At the same time, LMA fibers (with a core diameter $> 20 \mu$m) are not strictly single-mode, which adversely affects not only the quality of the beam, but also the stability of the single-pulse regime [59]. Moreover, such schemes utilize free-space polarization optics at the expense of environmental stability.

The most successful attempt of the YDFL cavity lengthening [23] resulted in the generation of $< 200$ fs dechirped pulses with energy increased to $\sim 20$ nJ at 12.5 MHz repetition rate (with 6 $\mu$m core fiber and free-space optics). Using a narrow (5 nm) spectral filter inside a resonator, it was possible to obtain generation at a repetition frequency of 3 MHz [60]. In this case, the pulse energy at the output of the oscillator was 15 nJ, and the duration after compression was 670 fs. In addition, the work focuses on the prospects of amplification of such pulses, since the radiation of a chirped pulse generator can be sent directly to an amplifier, without using a stretcher, preamplifier, and modulator. Jiang et al. [61] reduced the pulse repetition frequency to 2.3 MHz using an Er-doped fiber as the active medium, which also acts as an efficient spectral filter. They succeeded to reach acceptable environmental stability in 90-m long SMF with NPE mode-locking in spite of its extreme sensitivity. High long-term stability of repetition frequency and laser operation was observed for more than two weeks, although the NPE effect in a long SMF is very sensitive to external influences. The maximum pulse energy at the laser output was 9.4 nJ.

Interesting results have also been obtained in attempts to increase the resonator length without spectral filters. The creation of a high-energy femtosecond pulse
system with a repetition frequency of 4 MHz (resonator length 50 m) is reported in [62]. Here, the resonator of the master generator is made of SMF-28 telecommunication fiber with a core diameter of 8.5 \( \mu \)m. Mode-locking is provided by the NPE effect, and the active medium is Yb-doped GTWave double-clad fiber. The master generator has two outputs with a total energy of \( \approx 10 \) nJ per pulse. Kobotsev et al. [63] also increased the Yb laser length to a record value of 3.8 km, thereby reducing the repetition frequency to 77 kHz and increasing the pulse energy to 3.9 \( \mu \)J. However, this regime was no longer the regime of highly chirped pulses: the spectral width was only 0.35 nm and the duration of the generated pulse was 3 ns. The autocorrelation function (ACF) of such pulses has a two-scale structure with a narrow peak and a broad valley, which correspond to stochastic mode-locking for which pulse compression is problematic. As the authors showed later by numerical simulation [57], a transition from coherent (chirped) pulses to stochastic ones in the SMF based schemes with NPE mode-locking occurs at the fiber length of about 100 m that is agreed with experiments [61]. In most other experiments, this length is shorter (\( \approx 10 \) m) because of higher losses and higher sensitivity to environment increasing random birefringence of the fiber, see discussion at the end of the previous section.

As reported in [64], the problem of stochastic regime in such ultralong SMF cavities can be principally solved by using another mode-locking technique, for example, point-action saturable absorber based on single-wall carbon nanotubes. The authors use a synchronously scanning streak camera and a monochromator to directly measure the pulse spectrogram. This measurement showed that the pulses have a relatively clean temporal profile, and its average chirp appears to be nearly linear in a broad range thus demonstrating a principal possibility of so-called giant-chirp oscillators. However, the authors note that it is not possible to compensate for the chirp over several nanoseconds pulse duration with any practical compression scheme. Moreover, the chirp characteristic has a significant spread in each point of about 0.1 part of the whole frequency range; therefore, it seems hardly possible to compress the pulses by more than 10 times.

Other ways to improve stochastic regime in ultralong cavities include more complicated cavity design [65] and implementation of active mode-locking technique [66] in Er-doped fiber laser. In [65] pulse energy of nearly 1.7 \( \mu \)J is obtained at repetition rate is 35.1 kHz in a fiber laser with a linear-ring cavity. In [66], erbium fiber laser generates chirped pulses with energy 34 nJ at 163.8 kHz repetition rate.

Endeavors to improve environmental stability involved also a use of PM fibers in combination with saturable absorber mirror (SAM) [67] demonstrating generation of 1 nJ pulses at 17 MHz rate. In similar linear scheme with PM fibers and semiconductor SAM [60], pulse energy exceeded 2 nJ, but in both schemes dechirped pulses are longer (\( \geq 300 \) fs) than in NPE-based lasers because of long SAM relaxation time. In addition, all the discussed schemes utilize free-space optics that degrades both the system stability and limits possibilities of its practical use.

For practical applications of a laser system an all-fiber design is the most desirable, that is also true for femtosecond lasers. All-fiber NPE-based Yb oscillator delivering 1.8 nJ, 179 fs dechirped pulses at 33 MHz repetition rate
was demonstrated by Schultz et al. [68]. The same group later developed an all-fiber Yb laser with higher (3.6 nJ) pulse energy [69]. Recently, good results were demonstrated with the use of NOLM [70] and NALM [71] (nonlinear optical or amplifying loop mirror, correspondingly) mode-locking techniques in an all-PM fiber cavity providing exceptional environmental stability. In [71], the authors develop an all-PM all-fiber giant chirp oscillator and present an evident possibility of energy upsampling by cavity lengthening. At the maximum cavity length $L = 100$ m, laser generates stable chirped pulses with energies of 16 nJ that can be compressed to 370 fs duration. At further cavity lengthening, the authors observed a strong Stokes signal induced by SRS and significant stability reduction of a laser generation. They interpreted this fact that SRS is the main reason of the stability break treating this mechanism as a fundamental limitation of energy scalability [72].

Note that in the hybrid SM–PM fiber oscillator free of NPE-overdriving discussed at the end of Section 12.2.3, proposed and realized in earlier papers [28, 29], the SRS effect is observed at comparable pulse energies achieved in shorter cavity, but it does not break the pulse stability even at higher pulse energies. This effect limits the DS energy above the SRS threshold converting all the excess energy into noisy pulse generated at the Stokes-shifted wavelength, but without stability break. The experimental realization of such an all-fiber oscillator together with results on its output parameters and further scaling limitations are presented in the next section.

12.3.2

SRS Threshold for Dissipative Solitons at Cavity Lengthening

The basic design of the DS all-fiber laser is depicted in Figure 12.5. This YDFL scheme is similar to conventional ring schemes of NPE mode-locked DS fiber lasers [13, 19, 20, 23, 24, 73]. A key difference is that the cavity is divided into two parts: very long passive PM fiber part (blue on the scheme) and very short
12.3 Scaling of Dissipative Solitons in All-Fiber Configuration

SMF part (red on the scheme) between polarization controller (PC) and PBS. Such configuration was proposed for elimination of NPE overdriving (see Section 12.2.3) thanks to the possibility to control a rotation angle of the polarization ellipse by varying the length of SMF part of the cavity. The first realization of scheme [29] has confirmed the validity of this concept demonstrating stable mode-locking in a 30-m fiber cavity. In addition to mode-locking, the SMF part is also responsible for the pulse amplification, balancing the losses on the PBS output and spectral filtering naturally defined by the Yb$^{3+}$ gain bandwidth and the spectral function of WDM$_1$ (see [29, 30] for details). Moreover, since the SMF part is much shorter than the PM part, the effects of mode-locking, gain, and filtering may be treated as point-action, while DS evolution defined by nonlinearity, dispersion, and the studied SRS effect is distributed along the PM fiber. It is an important assumption that is used for further simulations in Section 12.4.1. Mention that a cloud-inset on the scheme corresponds to the modification that allows us to convert a noisy Raman pulses to a new type of the solitons – RDSs. This opportunity is discussed in detail in Section 12.4.3.

The SMF part of the cavity with optimal length $L_{\text{SMF}} \approx 1.5$ m consists of non-PM SMFs: passive fiber (Nufern 1060-XP) and a 15 cm piece of active Yb$^{3+}$-doped fiber (CorActive Yb-17-05) pumped by 976 nm single-mode LD via 976/1030 nm wavelength division multiplexer (WDM$_1$). Small-signal Yb$^{3+}$ gain is $\sim 25$ dB at spectral maximum ($\sim 1025$ nm). A PC and a PBS set before and after the SMF part provide NPE-driven self-amplitude modulation. The DS part consists of only PM fiber components: long section ($L_{\text{PM}} = 25 – 115$ m) of passive PM fiber (Nufern PM980-XP), a PM isolator, and a 1% PM splitter, that is used for a monitoring the intracavity parameters of the generated pulses. The core diameter of the PM fiber used is $d \approx 5.5$ m. The dispersion coefficient measured by the fiber white light interferometer has the value of $\beta_2 = 0.023$ ps$^2$ m$^{-1}$. At the outputs, we have some additional fiber splitters that are not present in Figure 12.5 being used for simultaneous measurements of power, radio frequency (RF) and optical spectra. All outputs of the laser were protected by angled physical contact (APC) connectors.

For a pulse compression, we use a standard double-pass compressor consisting of a grating pair (not shown in Figure 12.5 for simplicity). The pulse width is measured by scanning interferometric autocorrelator (Avesta AA-20DD) with input pulse duration ranges from 10 fs to 6 ps and by FROG (produced by Mesaphotonics) with the extended range up to 100 ps (hand made by means of an additional motorized translation stages). The pulse spectrum is characterized by conventional optical spectrum analyzer (Yokogawa 6370). Pulse train and radio frequency beating spectrum were monitored at the same time by 1-GHz photodiode on the oscilloscope and on Agilent N9010A spectrum analyzer, correspondingly.

A linearly polarized pulse is equally affected in PM fiber by dispersion and self-phase modulation providing large linear chirp [28]. So, the fiber laser exhibits generation of stable pulses in all the studied range of the cavity lengths with their duration 30–70 ps nearly proportional to cavity length. The key advantage is that the lengthening of PM-part does not lead to the stability loss: the total cavity
length can be easily increased from $L \sim 30$ m to $L \sim 120$ m (repetition rate goes down from $\sim 7$ to $\sim 1.7$ MHz). However, already at the $L = 30$ m (that is about two times more than in Chong et al. [23]) another limiting factor comes into play, namely SRS. In addition to the main DS peak (centered at 1015 nm), a Raman peak (centered at 1060 nm) appears in the generated spectrum (see Figure 12.6). It becomes stronger for longer cavities, where a noisy RP develops with an energy comparable with that for a DS. This effect limits DS maximum energy without deteriorating its stability: RF spectra measured at $\sim 750$ MHz remain narrow ($\sim 1$ kHz) and are of high contrast ($\sim 60$ dB, see insets in Figure 12.6) in all studied range. For a 30 m cavity length, the maximum pulse energy achieved is 23 nJ (> 150 mW output power at 390 mW pumping), 17 nJ of which corresponds to the DS. To the moment of publication [29], this result was about one order of magnitude higher than the pulse energy in previous all-fiber YDFL configurations [68, 69], and comparable with the best results for SMF-based ANDi laser with bulk optics [23]. Notably, we used fibers of smaller diameters together with lower pump powers.

To our surprise, we have succeeded in the generation of stable DS pulses at lengths up to $L = 120$ m (corresponding to the repetition rate $F \sim 1.7$ MHz). At $L = 60$ m and $L = 90$ m, the pulse energy reaches the maximum value of 27 nJ (23 nJ of which corresponds to the DS). At the same time, the energy of intracavity DS pulse remains constant at different pump powers. In longer cavity (120 m), the output DS energy slightly decreases. Nevertheless, at all cavity lengths the optical spectrum has a shape characteristic for a highly chirped DS (with steep edges, see Figure 12.6b). More detailed discussion on intracavity and extracavity pulse energies can be found in [74].

For all cavity lengths, the ACF for both chirped and dechirped DS pulses has been measured. The intensity autocorrelation trace has triangle shape, see Figure 12.7a, that corresponds to the rectangular pulse shape of the same duration, in accordance with theoretical predictions [26, 27]. In terms of analytic solution for highly chirped DS [26, 27], this domain is characterized by a single composite parameter $R$ tending to zero. However, the DS spectrum has

**Figure 12.6** The measured optical pulse spectrum (in – inside the cavity at 1% port, out – out of the PM splitter): spectrum for a maximum DS output energy for 30 m and 90 m cavity length is present on (a) and (b) correspondingly.
12.3 Scaling of Dissipative Solitons in All-Fiber Configuration

Figure 12.7 Autocorrelation functions: (a) by intensity for chirped pulse at different length of cavity and (b) interferometric for compressed pulse at 30 m and 90 m cavity length.

fluctuations that make difficult its comparison with the DS resonance shape (e.g., see [19]), but it definitely has steep edges with the corresponding ripples in the autocorrelation trace (Figure 12.7) after the compressor. The output pulses were externally compressed from 30–70 ps down to 200–300 fs (at a transform-limited pulse duration of 160 fs for 30 m cavity length); see Figure 12.7b. So, the resulting compression factor is as high as \( \sim 150 \) at \( L = 30 \) m and is more than 200 for \( L = 90 \) m.

The auto- and cross-correlation traces of the DS and the Raman pulse indicate different chirp, although they copropagate in the laser cavity and have similar durations, with a \( \geq 50 \) ps shift between their centers, that increase linearly with cavity length. Note that the importance of the Raman effect at the pulse chirping was first predicted in theoretical study by Schadt and Jaskorzynska [75]. Appearance of the Raman component in a high-energy fiber laser was mentioned without details and discussion by Kobtsev et al. [62].

Let us estimate a threshold energy for the Raman conversion of DS pulses. In the Raman process, an amplification of the wavelength-shifted Stokes wave from the noise level is induced by the DS pulse field. Therefore, the Stokes wave starts propagating together with the DS pulse and its power grows exponentially with the coefficient \( g_R P_m \), where \( P_m \) is the DS peak power and \( g_R \) is the Raman gain coefficient normalized on the mode field area. Because the DS and Stokes pulses have different group velocities \( (v_0 \) and \( v_R \) correspondingly) due to dispersion, the Stokes pulse “overpasses” the pump pulse (DS soliton) of duration \( T \approx 30 \) ps after propagation of critical distance \( L_C = T/\delta v^{-1} \sim 20 \) m, where \( \delta v^{-1} = v_0^{-1} - v_R^{-1} \approx 1.5 \) ps m\(^{-1}\). Outside the interaction region, DS power tends to zero and the Stokes radiation starts to attenuate. So, the maximum Stokes power is shifted relative to the main DS pulse. We can estimate the maximum peak power of the Stokes pulse as

\[
P_R = P_\text{noise} \exp \left( g_R P_m T/\delta v^{-1} \right).
\]

Evaluation of the noise power that takes into account about a four-orders-of-magnitude difference between the pulse duration and the round-trip time, resulting in \( P_\text{noise} \sim 10^{-8} P_m \), leads to the ratio for the peak powers and thereby to
the following ratio of the Raman and DS pulse energies:

$$\frac{E_R}{E} \propto \frac{P_R}{P_m} \approx \exp \left( \frac{g_R P_m T}{\delta \nu^{-1}} - 18 \right).$$

(12.18)

The Raman-component energy, $E_R \sim P_R T$, becomes comparable with the energy of the DS pulse, $E = P_m T$, at critical intracavity energy estimated as $E_{cr} \approx 18\delta \nu^{-1}/g_R$. This estimate corresponds to $\sim 10$ nJ for our 5.5 $\mu$m fiber with the Raman gain coefficient $g_R \approx 2.5$ W$^{-1}$ km$^{-1}$. Note that critical output energy in the experiment is about two times higher than the intracavity one [29].

The smaller the core diameter, the lower the critical intracavity energy at which Raman scattering becomes significant. Therefore, to push further the scaling of the all-fiber dissipative-soliton laser, one should combine our approach with LMA fibers [24]. The Raman effect in this case will be suppressed because of the lower coefficient $g_R$. In such a combined SMF-PM all-fiber configuration with a 25 $\mu$m core LMA, the estimation above gives $E_{cr} \sim 0.5$ $\mu$J. As additional steps in the direction of the energy upscaling, one can increase the dispersion parameter $\delta \nu^{-1}$.

Expression (12.17) can be also rewritten through chirp parameter $f$. Substituting Eqs. (12.1) and (12.6) and taking that $\delta \nu^{-1} \approx \Delta_{SRS} \beta_2$, we obtain

$$P_R = P_{\text{noise}} \exp \left( \frac{g_R \Delta^2 \beta_2 / 2\gamma \cdot 4f / \Delta}{\Delta_{SRS} \beta_2} \right) = P_{\text{noise}} \exp \left( \frac{2f g_R \Delta}{\gamma \Delta_{SRS}} \right),$$

(12.19)

where $\Delta_{SRS}$ is the Stokes shift in the fiber. Thus, the Raman gain induced by the DS depends mainly on its chirp parameter $f$ since $\Delta < \Delta_{SRS}$ and $g_R < \gamma$ in fibers. It means that the SRS effect becomes much more important for highly chirped DSs.

Influence of noise and SRS on the DS energy scalability can also be considered in terms of DS master diagram [49]; its representation in a two-dimensional parametric space [76] is shown in Figure 12.8. Without noise and SRS a dissipative soliton is perfectly energy-scalable. The zero-level iso-gain defining the threshold of DS stability against a vacuum excitation is shown by solid black curve in Figure 12.8 and demonstrates the DS resonance existence for a chirped DS: $\lim_{c \to 2/3} E = \infty$. A soliton is unstable to right of this curve. However, such a perfect scalability, for example, in the form of dissipative soliton resonance, can be broken by a noise amplification and an SRS. When a noise and SRS contribute the DS stability border changes drastically (compare black, pink, and blue lines in Figure 12.8) so that an asymptotically unlimited accumulation of energy becomes impossible and the so-called “dissipative soliton resonance” disappears.

Although the Raman scattering is a parasitic effect in the problem of femtosecond fiber lasers energy scaling, it will be especially interesting to study in detail the features of the generated Stokes radiation and its interaction with the DS, which will help to answer the question why such a strong Raman scattering does not deteriorate the soliton in such a long cavity. Theoretical and experimental study of the detailed evolution for the DS in long fiber cavities in the presence of strong SRS effect is presented in the next section.
Figure 12.8 Stability thresholds ($\sigma = 0$) of HCDS solutions of CQGLE without noise and SRS (solid black curve), with noise and no SRS (red dashed curve), as well as with noise and SRS (blue dashed-dotted curve). A sole HCDS exists below and left of the corresponding curves. The parameters of CQGLE are $\kappa = 0.1 y L$ and $\zeta = 0.05 y L$. The spectral filtering parameter $\alpha = 366 \text{ fs}^2$ corresponds to an approximately 40 nm bandwidth. Other details can be found in [76].

12.4 SRS-Driven Evolution of Dissipative Solitons in Fiber Laser Cavity

12.4.1 NSE-Based Model in Presence of SRS

In this section, we report on the comprehensive theoretical and experimental study of the new physical effects defining DS formation and evolution in the presence of strong SRS. We demonstrate that SRS acts not only as an additional channel of energy dissipation destroying DS, but can also support it enabling thus the generation of a stable “DS—Raman pulse” bound complex in all-fiber ring laser cavities with order-of-magnitude larger lengths (>100 m) than in conventional NPE-mode-locked DS fiber lasers [19, 20, 23, 24, 73]. We also discuss the influence of SRS conversion and noise influence on the DS energy scalability. As a result, we have identified the maximal reachable energy of DS with a cavity lengthening.

We study an YDFL experimental scheme that is described in Section 12.3 and depicted in Figure 12.5. As already discussed, the main feature of the scheme is that it provides stable (overdriving-free) mode-locking via NPE in a short SMF part, whereas the cavity length is increased independently by means of PM fiber. Consider first the case without the cloud-inset.
12.4.1.1 Model Details

For numerical modeling of a fiber laser, we take into account the discreetness of its intracavity elements. Thus, the effects of mode-locking, gain, and filtering in the short SMF are treated as point-action, while DS evolution inside the long PM fiber in the presence of strong SRS is studied with generalized NLSE; see [19, 20, 77] and citation therein. To include the new effect of the DS spectrum conversion via Raman effect (in the PM fiber the DS and Raman pulses are equally polarized that was directly checked), we add SRS term in the equation, similar to the case of super-continuum generation (see, e.g., [78, 79]):

\[
\frac{\partial A}{\partial z} = -i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + i \frac{\beta_3}{6} \frac{\partial^3 A}{\partial t^3} + i \gamma \left( A(z, t) \int_0^\infty R(t') |A(z, t-t')|^2 dt' \right),
\]

(12.20)

where \( A(z, t) \) is the electric field envelope, \( \beta_2 \) and \( \beta_3 \) are the second- and third-order dispersion coefficients at the central frequency \( \omega_0 \), \( \gamma \) is the Kerr nonlinearity coefficient, and \( R(t) \) is the Raman response function, which includes both instantaneous electronic and delayed Raman contribution. In our simulations, we use the multiple-vibrational-mode model for the Raman response described in [80]. The calculated peak Raman gain (with the Stokes shift \( \Delta \lambda = 45 \) nm) is nearly equal to the experimental value of \( g_R = 2.5 \) W\(^{-1}\) km\(^{-1}\) [29]. The specific fiber parameters that are used in simulations throughout the paper are as follows: \( \beta_2 = 22 \) ps\(^2\) km\(^{-1}\) and \( \beta_3 = 0.037 \) ps\(^3\) km\(^{-1}\), \( \gamma = 6 \) W\(^{-1}\) km\(^{-1}\).

The equation was solved by using the symmetrized split-step Fourier-transform method. The simulations are run until the pulse field reaches the steady state after a certain number of cavity round-trips, taking into consideration the contribution of the point-action WDM having a stepwise transmission spectrum, amplification in Yb\(^{3+}\)-fiber and the NPE-induced intensity modulation described later.

Amplification in an Yb-doped active fiber \( l \) with a Lorentzian line of a 40 nm bandwidth, a central wavelength of \( \lambda_0 = 1025 \) nm, and a small-signal gain coefficient of \( g_0(\lambda_0) \approx 170 \) dB m\(^{-1}\) was simulated in the spectral domain. The gain saturation is modeled according to

\[
g(E) = \frac{g_0}{1 + E/E_{sat}},
\]

(12.21)

where the total gain \( g_0 l \) equals to 25 dB, \( E = \int_{-T_R/2}^{T_R/2} |A(z, t)|^2 dt \) is the signal energy, \( E_{sat} = P_{sat} T_R \) is the saturation energy, and \( T_R \) is the cavity round-trip time. The amplifier noise is simulated by an additive complex “white” noise with 10\(^{-7}\) W average power, which results in spectral noise level –40 dB relative to DS power (Figure 12.11b).

Since the SMF part in our scheme is short compared with PMF, the angle of the NPE-induced polarization rotation in SMF is small. So, we can describe the action of the NPE-based modulator in a scalar form by the cubic–quintic nonlinear term for the pulse amplitude \( A(t) \) [39] and rewrite the nonlinear modulator transmission \( \rho \), depending on the incident power \( P = |A|^2 \) in the following form:

\[
\rho = \rho_{max} - \left( \frac{P}{P_{cr}} - 1 \right)^2 (\rho_{max} - \rho_{min}),
\]

(12.22)
12.4 SRS-Driven Evolution of Dissipative Solitons in Fiber Laser Cavity

Without feedback

With feedback

![Figure 12.9](image)

Figure 12.9 The calculated shapes of the generated pulses in corresponding points (A,B,C,D,E) of the scheme without and with the feedback loop shown in cloud-inset of Figure 12.5. The RP in box C (right) present before it attenuated by factor $R$.

![Figure 12.10](image)

Figure 12.10 Evolution of the pulse shape (a) and spectrum (b) along PMF of $L = 30$ m in the scheme of Figure 12.5 without feedback loop.

where $\rho_{\text{min}} = 0.05$ characterizes the minimal transmission at low powers and $\rho_{\text{max}} = 0.5$ is the maximum transmission at the critical power $P_{\text{cr}} = 850$ W, which corresponds to the experimental data for the PBS splitting ratio in the DS regime.

12.4.1.2 Simulation, Comparison with Experiment

The numerical results have been compared with the experimental data obtained. First, such a comparison was made for $L = 30$ m.

Calculated pulse shapes are shown in Figure 12.9 (left column) in different points of the cavity. Stationary pulse evolution along the cavity is shown in Figure 12.10. A high-power (> 500 W) DS with a duration of 30 ps at point A ($z = 0$) propagates without significant changes of the shape, but its intensity is
I
300

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Figure 12.11 Calculated and measured pulse shapes (a) and spectra (b) and at the output port E in scheme of Figure 12.5 without feedback loop. Calculated and measured ACF and CCF traces are present in inset.

decreased down to 350 W (z = 30 m) due to generation of a noisy RP (represented by the lower stripe of “<”-shaped trace in Figure 12.10a). The DS and RP become fully separated due to dispersion at point B (z = 30 m), with spacing between the centers up to 50 ps. Pulse shapes at points A and B corresponding to the entrance and exit points of the PM fiber are also shown in Figure 12.9. Then, in a short SMF the pulses are filtered and amplified in an Yb$^{3+}$ fiber. The power of the DS pulse reaches ~1 kW while amplification of the Stokes pulse is much smaller as its spectrum is far off the Yb$^{3+}$ gain maximum. The amplified pulse is then equally divided by (a) PBS dumping the counter-polarized part (Figure 12.9, panel E in left column) out of the cavity and (b) sending the co-polarized part back into the PM fiber. The DS evolution then starts again at point A (z = 0) of the PMF. Note that the residual part of the RP generated on the previous round-trip enters the PMF together with the DS (the weak lower stripe in Figure 12.10a and the relevant forerunner (left) pulse in Figures 12.9 and 12.11a).

The corresponding spectra of the DS (centered at 1015 nm) and the RP (centered at 1060 nm) (Figure 12.10b) do not vary substantially and demonstrate significant noise of the RP. The calculated output spectra (port E in Figure 12.10) for the PMF length 30 m are compared in Figure 12.11b with the experimental ones demonstrating their good agreement. The corresponding pulse shapes in the time domain are also given here. One can see that the spectral structure realized in the experiment and numerical simulation does not correspond to that of a plain DS in the absence of SRS (dashed curves in Figure 12.11b, $f_R = 0$). Thus, the blue shift of the DS spectrum can be attributed to the SRS, which “eats” a red spectral part of DS. The spectral shift is limited by the WDM$_1$ loss increase at < 1005 nm.

The corresponding temporal pulse profiles calculated with and without the SRS term are shown in Figure 12.11a. The SRS-driven DS has 1.5-times higher power and 3-times shorter duration than the Raman-free DS. This means that the SRS effect leads to strong temporal–spectral filtering of the DS, resulting in its specific shape.

In the experiment, ACF and cross-correlation function (CCF) were extracted from experimentally measured FROG trace [81]. The experimental results shown
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Figure 12.12 Energy of the DS in the experiment (green triangles) and simulation (green solid line). Calculated total energy for the DS and RP (dashed line).

The experimental point obtained for $L = 120$ m ($E \approx 20$ nJ) is not added because the pump power was reduced due to lower stability at this length.

in the inset of Figure 12.11 agree with the calculated ones: the triangular ACF confirms the rectangular DS shape, and the CCF shows the 50-ns delay of RP relatively to DS. The second peak of CCF (and third peak in time domain correspondingly) is an RP from previous round-trip. The FROG trace reveals partial coherence between DS and RP pulses that was also confirmed by a possibility to compress RP by 3–4 times [30].

The numerical results also confirm the experimental results of [29] so that the upper limit for the maximum DS energy was revealed for a 60-m-long fiber oscillator (Figure 12.12). The limit is defined by the SRS converting an excess energy of the DS to the RP. The energy of the RP becomes comparable to the DS level, but, surprisingly, such efficient conversion does not destroy a stable DS. For cavities up to 90 m, the DS energy limit defined by the excess energy conversion into the RP is lower than the limit defined by destabilization of the DS by increasing noise. The maximum RP energy is also limited because of its sufficient attenuation and generation of higher-order Stokes pulses. So total energy of the DS–RP complex in the experiment is also saturated at $\sim 30$ nJ level that is confirmed by the calculations (see Figure 12.12).

Therefore, we demonstrate that the stable DS pulses can be generated in a fiber oscillator in spite of strong SRS, in contrast to the conclusions of [72]. The generated noisy RP does not destroy the DS and can even form with it a stable bound complex. In this complex, the DS provides amplification for the RP, whereas the RP stabilizes the DS energy acting as a temporal and spectral filter for DS. Intuitively, the Raman process is incoherent and acts in our case as an additional energy loss channel. The DS stability cannot be affected by the RP, at least if we compensate the loss by proportional pumping. The pulse energy realized in our experiment is higher than that obtained in other all-fiber schemes in the absence of SRS, though
SRS-induced energy conversion defines an upper limit for the DS energy and substantially changes the DS energy scalability.

In the next section, we show that the RP noise seen in Figure 12.11, can be successfully suppressed by the feedback loop, shown as a cloud-like inset in Figure 12.5.

12.4.2
Generation of Stokes-Shifted Raman Dissipative Solitons

In the previous section, we demonstrated that SRS in the fiber laser converts the excess energy out of the coherent DS to a noisy Stokes-shifted RP thus limiting the maximum DS energy [29, 30]. The RP noise is ascribed to the fact that it starts from spontaneous emission reaching high energy after only one round-trip of the DS, which acts as a pump for the RP. Because of the difference in group velocities, the RP afterwards runs away from the DS.

In this section, we study a possibility to initiate the Raman pulse not from noise, but from a seed pre-pulse provided by a feedback for the RP at proper timing defined by the group velocity dispersion (see cloud-inset on Figure 12.5). As a result, we have discovered that a feedback provided by re-injection of an RP into the laser cavity leads to its transformation into a coherent RDS. Together, DS and RDS form a stable complex of higher total energy and broader spectrum than those of the DS alone.

If the pre-pulse is re-injected into a laser cavity with the proper amplitude, it will then be synchronously amplified by the DS circulating in the cavity, similar to synchronous pumping mode-locking technique [79, 82]. The difference in the group velocities of DS and RP is compensated by the bypass fiber of the feedback loop providing a necessary delay for the RP.

To test the effect of the RP feedback on the pulse structure, we first did numerical simulations with the generalized NLSE 12.20. The pulse shapes calculated with feedback are shown in the diagrams of Figure 12.9. Evolution of the pulse temporal and spectral shapes along the cavity is shown in Figure 12.13. A stable noise-free RP was found for the feedback coefficients $R = P_f/P = 10^{-2} - 10^{-6}$, where $P$ is the circulating light power with its small re-injected part $P_f$.

The RP and DS co-propagate together as one stable two-color complex. For oscillator cavities longer than 80 m, the complex becomes unstable. In the range of cavity lengths supporting the stable complex, increase of the pump power leads to a multipulse regime similar to conventional solitons [83–87]. Within each complex, the energy is nearly equally divided between the DS and the noise-free RP (see Figure 12.9, panel B in right column). The pulse evolution along the cavity (Figure 12.13) demonstrates how a stable noise-free RP is formed at the beginning of PMF due to conversion of the “red” part of the DS into the Stokes-shifted wavelengths around 1060 nm. Then, the DS and RP separate due to dispersion. The delay line (DL) in the feedback loop compensates the dispersion shift and after partial outcoupling through ports D and E and amplifying in Yb$^{3+}$ fiber, evolution starts again at point A. As can be seen (Figure 12.9), the feedback makes pulse
Figure 12.13 Evolution of the pulse shapes and spectra along the PMF and SMF (with the active Yb³⁺-doped part) sections in a scheme of Figure 12.5 with the Raman feedback loop comprising the delay line (DL) shown as the white box, with parameters similar to that in the experiment. Points A, B, C, D, E, and PBS (polarization beam splitter) of the scheme of Figure 12.5 are marked at the corresponding distances [31].

Figure 12.14 Pulse shapes: (a) the simulated feedback-defined evolution of the intracavity pulses in the scheme of Figure 12.5 (point B) versus the round-trip number. The zero time offset is bound to the position of the main dissipative soliton. (b) The calculated DS and RDS pulse shapes inside the cavity (point B in Figure 12.5) and their instant frequencies. Inset: the dechirped DS–RDS complex with 70-ps delay compensation in the Raman feedback loop [31].

Figure 12.14a shows the transition from noise (the pre-pulse) to the steady-state pulse. In the initial state of the laser, amplification of noise starts at both fundamental and Raman wavelengths. Then, two chirped pulses are formed almost simultaneously. The resulting complex of two bound dissipative solitons (the main and Raman ones) circulates in the cavity in an extremely stable manner. The temporal shapes of stationary DS and RP are depicted in Figure 12.14b. Analysis of the RP spectral and temporal shape together with its spectral phase shown in Figure 12.14b led us to the conclusion that the noise-free RP realized numerically in the scheme shown in Figure 12.5 can be called a RDS.

Based on the detailed comparison, we can conclude that the RDS realized in [31] has the same equilibrium characteristics and demonstrates similar evolution basically different (compare also Figure 12.10 with Figure 12.13). The noise is eliminated and the energy becomes concentrated in well-defined temporal and spectral ranges.
dynamics as the chirped pulses generated in the ANDi laser configuration agreed to be dissipative solitons [14, 19, 88]. In addition to similar characteristics of the RDS and DS: temporal and spectral shapes, chirp and extracavity compression factors as well as their sensitivity to spectral filtering, the dissipative soliton nature of the coherent linearly chirped Raman pulse was also confirmed by a possibility of its description in the frame of CQGLE (see [31] for details).

We presented above all the basic arguments including the solutions of the two main equations in order to formulate unambiguously that the new stable Raman-shifted pulses are dissipative solitons. A fair comparison of NLSE and CGLE is a very demanding task, and we made only the first steps in this direction. The approximation of the uniformly distributed cavity relevant for CGLE is rather rough for any laser generating dissipative solitons, since typical amplitude variation at point-action dissipative sections (gain, filtering, outcoupling, etc.) of the laser cavity amount to about 1 order of magnitude, while temporal and spectral widths may exhibit much smaller changes; see [19, 20] and citations therein. The same is true for the new dissipative soliton generated via Raman process, where the amplitude variations are even stronger because of much higher Raman gain and lower feedback coefficient. The pulse widths are also slowly varying here, similar to that for conventional DS. We call a new soliton centered at 1060 nm a Raman DS in order to emphasize its origin leading to quantitative difference of the pulse dynamics from the main DS at similar qualitative behavior. The average characteristics of the DS and RDS are nearly the same and close to the dissipative soliton solutions of CGLE.

12.4.2.1 Proof-of-Principle Experiment

To verify such promising theoretical predictions, we realized experimentally the scheme shown in Figure 12.5 with feedback loop. The data presented in Figures 12.15 and 12.16 show that the pulses realized in the experiment are basically different from those observed without feedback, but in good agreement with theoretical predictions. The spectral division between output ports in the experiment (∼10 dB) is not as high as in the simulations (> 40 dB), and (ii) the RDS port has significant spectral ripples around 1010 nm [31].
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Figure 12.16 Autocorrelation traces and radio frequency beating spectra of the realized pulses. The measured (points) autocorrelation traces of the DS (a) and RDS (b), and the calculated (lines) for the 40-m cavity oscillator with the feedback coefficient $R = 10^{-4}$. Inset: the corresponding FROG traces; Interferometric ACF traces shown for DS (c) and RDS (d) after a double-pass compressor consisting of a grating pair. The compressor introduces one-pass group delay dispersion of $D \approx -0.7$ ps$^2$. Insets: Radio frequency spectra measured at a repetition rate of 5 MHz with 1 Hz bandwidth [31].

with the RDS predicted by simulation. The RDS’s spectrum has a typical shape with steep edges, similar to the DS. The FROG trace (Figure 12.16) shows that the phases, pulse durations, and shapes of the DS and RDS are similar. The duration of the dechirped DS and RDS is measured to be 250–300 fs and 300–400 fs, correspondingly; see Figure 12.16c and d. The compression factor for the RDS is better than 100, that is, ~30 times higher than that for the noisy RP in [29, 30], and only slightly lower than that for the DS. The measured interferometric ACF for the RDS has a contrast close to 8:1 (Figure 12.16c and d), only slightly deviating from that for the dechirped DS. A radio frequency beat signal measured at the fundamental repetition rate and its 150th harmonic are also similar for the DS and RDS exhibiting a high contrast in the range of 60–100 dB (see insets in Figures 12.16c and d). These values are comparable with those for the ~10 nJ SRS-free DS generated in all-fiber configurations of Yb-doped fiber oscillators (see, e.g., [70]) and only slightly worse than the best results for the ~3 nJ Er-doped fiber oscillators [11]. In our case, the total pulse energy at the DS and RDS outputs is 25 nJ, with nearly equal division inside the cavity and a fourfold difference at the outputs because of selective DS amplification.
The first-order RDS described earlier substantially differs from a classical Raman soliton, which is a version of fundamental conservative soliton and appears as a result of the Raman frequency shift in the anomalous dispersion range [79, 82], for example, in transmission lines [89]. As the RDS is bound with the DS via a nonlinear Raman process, they are also different from one-color (i.e., of the same spectrum) bound solitons, or multisoliton complexes based on Kerr nonlinearity [18, 83–87]. In our case (see Figure 12.14b), the DS and RDS form a stable two-wavelength chirped DS complex that could be de-chirped into a single pulse under certain conditions. The fringe pattern in Figure 12.14b (inset) results from merging of the compressed coherent DS and RDS pulses behind the compressor. The fringe separation (~40 fs) is defined by the whole spectral range of the solitonic DS + RDS complex, whose compressibility can be similar to the cases of nonlinear spectral broadening in fiber [79] or a Raman-active mode-locked laser [90]. It is worth noting that the fringe visibility, which shows the coherence quality, depends on the feedback, with the highest coherence reached for the values $10^{-4} < R < 10^{-2}$. The results of simulation also show that the coherence of the RDS and DS is immune to noise variations in a broad range.

We also found that the shape of RDS is very sensitive to parameters of linear spectral filters, similar to that for conventional dissipative solitons. More detailed analysis of RDS characteristics depending on cavity parameters is presented in the next section.

12.4.3

Characteristics of Raman dissipative Solitons

In this section we analyze how the optical spectra of DS and RDS are sensitive to variations of cavity parameters: linear and nonlinear (SRS-induced) filtering and the Raman feedback loop.

12.4.3.1 Variation of the Soliton Spectra with Filter Parameters

Linear filtering is provided by two WDM couplers, whose spectra are sensitive to temperature and can vary in the range of several nanometers. The first filter (marked as WDM₁ in Figure 12.5) is a self-made PM-fiber coupler based on the internal Lyot filter consisting of a properly oriented short PM-fiber piece and a PBS. Such a WDM coupler/splitter has a typical sinusoidal spectral function (Figure 12.17a) providing the wavelength division 1015/1060 nm for two output coupler arms. The second coupler (WDM₂ in Figure 12.5 of the paper) is a commercial one having a stepwise transmission spectrum function with the cutoff at 1005 nm. The influence of the filtering effect of WDM₁ on the DS is small, but nonlinear filtering becomes important due to strong Raman conversion, similar to that in the case of a noisy Raman pulse observed without WDM₁ and delay loop in [30]. As follows from our simulations, the SRS effect “eats” the long-wavelength wing of the DS spectrum thus pushing the soliton spectrum out of the gain maximum (1025–1030 nm) toward the WDM₂-induced cutoff at 1005 nm. This effect defines the short-wavelength edge of the generated spectrum.
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By numerically varying the exact position of the spectral maximum of the WDM$_1$ filter (Figure 12.17a), we found that the RDS spectral width is very sensitive to these variations. As one can see in Figure 12.17b that a shift of only 5 nm (the minimum value we can control in the experiment) leads to a significant change of both the RDS position and width. Because the RDS spectrum obtained with the 1015/1060 nm filter fits better to the transmission function of the WDM$_1$ coupler measured under the experimental conditions (Figure 12.17a), we used these data for comparing the calculated and experimental spectra (Figure 12.15).

12.4.3.2 Variation of the Soliton Spectra with the Raman Feedback Parameters

The maximum intracavity energy of the dissipative soliton depends on the feedback value $R = P_f/P$ logarithmically according to the equation for the SRS threshold energy, $E_{th} \approx \ln(1/R)\delta v^{-1}/g_R$, where $\delta v^{-1} = v_{DS}^{-1} - v_R^{-1}$ is the inverse group velocity difference of the DS and RDS, and $g_R$ is the Raman gain coefficient. The threshold energy for the experimental parameters is estimated as $E_{th} \approx 10$ nJ. Variation of the soliton spectral shape with the feedback coefficient in experiment and simulation is presented in Figure 12.18. At the value $R < 10^{-6}$, the re-injected power $P_f$ approaches the energy of the spontaneous emission at the Stokes-shifted wavelength $(P_{sp}/P \approx 10^{-7})$, and the feedback becomes insufficient to overcome noise. At $R > 10^{-2}$, the single DS + RDS complex splits into two similar pulse complexes separated in time. Stable pulses in 40-m cavity are realized for $R = 10^{-2} - 10^{-6}$ and delays in the range 50–80 ps (with optimum at ~70 ps).

12.4.4 Generation of Multicolor Soliton Complexes and Their Characteristics

The scheme shown in Figure 12.5, with additional feedback loops can be used for generating next-order Stokes pulses of similar spectral and temporal shapes.
Figure 12.18 Intracavity spectra in experiment and simulation corresponding to different feedback coefficients. The spectra are measured at point B of the scheme of Figure 12.5.

We numerically realized a scheme with two feedback loops, with the corresponding results shown in Figures 12.19 and 12.20. Figure 12.19 illustrates the evolution of the chirped pulse in the stable three-color regime. To realize it, we made the following modifications of the scheme: (i) long ($L \approx 80$ m) PM fiber, (ii) two feedback loops, each of which comprises WDM$_1^{(i)}$ splitter and 1 : R$_{(i)}$ coupler (similar to the feedback loop in Figure 12.5) optimized for the first and second Stokes pulses ($i = 1, 2$): $R_{(1)}^{(i)} = 10^{-4}$, $R_{(2)}^{(i)} = 10^{-2}$. The regime becomes stable when we use, instead of sinusoidal WDM$_1$ (see Figure 12.17a), two special WDM$_1^{(1)}$ and WDM$_1^{(2)}$ splitters of a stepwise spectral function at cutoff wavelengths at 1035 nm and 1085 nm, correspondingly. An implementation of such “hard” filters eliminates the remaining small-amplitude parts of the Raman pulse, which cannot be fully suppressed in the case of a “soft” (sinusoidal) WDM$_1$ splitter (see Figure 12.13). Since each feedback loop operates independently, the scheme
12.4 SRS-Driven Evolution of Dissipative Solitons in Fiber Laser Cavity

Figure 12.19 Evolution of the pulse shapes and spectra along the PMF and SMF sections in a scheme of Figure 12.5 in the case of two feedback loops providing delay and filtering for the first and second Stokes waves with the additional output port $F$.

Figure 12.20 Three-color bound solitons. Results of simulation for the generated spectra in an 80-m-long PM-fiber cavity oscillator with the Raman feedbacks for the first-order Stokes ($R_1 = 10^{-4}$) and the second-order Stokes ($R_2 = 10^{-2}$) waves, in the presence of a band-stop filter at 1025 nm (10 nm width) and the following parameters of the $i$th feedback loop: delay time 140 ps ($i = 1$) and 280 ps ($i = 2$), stepwise WDM couplers with cutoff wavelengths at 1035 nm ($i = 1$) and 1085 nm ($i = 2$). It offers a way to generate next Stokes orders by adding corresponding feedback loops in a routine manner.

A number of optimization steps remain to be made in order to realize this regime experimentally, including filter optimization. The open question is whether one can realize experimentally first- and second-order RDSs with the equidistant spectra shown in Figure 12.20. In this figure, one can see also higher-order noisy pulses at $>1150$ nm as well as anti-Stokes components at $<1000$ nm generated in frequency mixing processes.

It is significant that not only SRS, but also other nonlinear processes may be explored for generating broadband multicolor dissipative soliton complexes. As demonstrated for the femtosecond solid-state lasers [90, 91], the combined action of SRS and four-wave mixing allow intracavity spectral broadening similar to the phenomenon of supercontinuum generation in fibers [92]. Successful
pulse synthesis based on the total phase and spectrum control was recently demonstrated for several synchronized few-cycle lasers operating in different spectral ranges [93] and for the light field synthesizer dealing with a coherent supercontinuum from a single few-cycle laser [94]. Based on our approach, intracavity pulse synthesis looks feasible, with an important advantage: it will be intrinsically stable.

12.5 Conclusions and Future Developments

An important class of the soliton solutions of Ginzburg–Landau equation, so-called HCDSs, is extensively analyzed in this chapter. Strongly inhomogeneous phase across such solitons characterizes their capacity to redistribute energy without loss of stability. This property provides energy scalability of HCDS, feasible in femtosecond laser oscillators either with the cavity length or active medium volume. The physical limitations of these steps are shown in the example of all-fiber femtosecond fiber laser oscillators based on nonlinear polarization evolution mode-locking mechanism. The reason of the stability break is an excessive rotation of polarization ellipse along the cavity [28]. The use of polarization-maintaining fibers allowed pushing further the cavity length limit. A laser configuration based on long PM and short SM parts resulted in a total cavity length of up to 120 m, with the following DS parameters: pulse energy > 20 nJ, the chirp parameter > 200 and a compressed pulse duration of 200–300 fs [29].

So-called master diagrams allow analysis of a complex laser operating in the positive dispersion regime by decreasing the number of efficient system parameters of the Ginzburg–Landau equation. Based on master diagrams, it is shown that laser noise is another factor influencing the pulse energy of the laser [76]. Being in its infancy, noise limitations of the maximal pulse energy are to be included in forthcoming descriptions of high-energy femtosecond lasers. Technically, the problem consists of determining the type and amplitude of noise.

For long laser cavities, a new limiting factor comes into play, namely SRS. The detailed analysis of the SRS-driven evolution of DS has been performed in frames of NSE in which the SRS term is added and the model has been compared with the experiment [30]. It appears possible to realize a stable DS in the presence of strong SRS. In general, the intrinsic Raman effect occurring in femtosecond fiber laser broaden our vision of the processes responsible for the pulse formation. One of its consequences is the appearance of a noisy Raman pulse linked to the main DS. On one hand, because of high Raman gain, the noisy RP can lead to the energy limitations of the main dissipative soliton. In principle, one or more RPs can reach energy comparable to that of the main DS. On the other, being properly treated, RPs nevertheless allow to scale the cavity length and pulse energy up to high extend.

The pulse structure becomes quite different in case of feedback that delays a noisy RP at its every cavity round-trip [31]. Being properly built, such feedback
leads to formation of a RDS instead of a noise Raman pulse in the feedback-free case. The optimal feedback is very weak \( R \approx 10^{-4} \) but it is enough to convert noise into the coherent pulse due to high Raman gain. In this sense, it is similar to the weak random distributed feedback based on Rayleigh backscattering converting Raman amplified noise in fibers into the CW laser radiation [95, 96].

The results of numerical simulations of both DS and DS–RDS lasers based on Schrödinger and Ginzburg–Landau equations are consistent with each other and with the experiment. At high laser pump, the RDS energy can reach the energy of the main DS. The RDS occupies another spectral range thus reducing the “energy competition” with the DS. At higher pump and in the presence of second feedback, second-order RDS can be generated. RDSs are coherent clones of the DS, with the residual noise comparable to the DS. As a result, they can be compressed externally from tens of picoseconds to hundreds of femtoseconds pulses, similar to the DS. Though a laser with more than 100 m cavity is realized, the upper limit in terms of the pulse stability is still not reached.

Technically, the RDS approach reviewed in this book chapter allows for its straightforward extension to multi-RDS realization based on new fibers and fiber components. In principle, the approach offers a possibility of increasing the intracavity complex energy by generating RDSs of different polarizations in other cavity loops having a common part with the main cavity. By the use of different fiber materials, one can vary spectral positions and spectral widths of RDSs. By having a multi-RDS soliton complex with spectrally equidistant soliton components and using a combination of SRS and four-wave mixing, a smooth ultrabroadband spectrum looks feasible for its further compression in order to generate single pulses of tens of femtoseconds and shorter.

Unique features of the soliton complexes demonstrated in the all-fiber scheme such as high pulse energy, broad spectral range occupied by the complex, coherence of the solitons in the complex, offer new challenging applications. In medicine, coherence optical tomography and nonlinear or multiphoton fluorescence microscopy [97, 98] can benefit from broad high-power coherent spectrum. In both cases, high stability of the solitons will allow for collecting intrinsically weak signals from scattered light. Near-future high-bit rate periodic transmission lines with Raman pump can use more broadband and quiet laser sources based on soliton complexes demonstrated in this article. High power and low noise [89] will allow for longer passive fiber pieces between the amplifiers. Kerr nonlinearity could be considered as the mechanism that stabilizes the dissipative solitons in the complex. Alternatively, specialty fibers with properly designed dispersion for the main and frequency-shifted dissipative solitons can be used.

Multicolor dissipative soliton complexes offer further extensions for frequency comb spectroscopy [92, 99] and optical parametric amplification schemes (broadband seed) [100]; schemes for generating new spectral components in several synchronized enhancement cavities [101] containing a common nonlinear element, when each cavity is seeded by a certain RDS. Intrinsic stability and coherence of the complex will lead to a stable operation of the bundled enhancement cavities.
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