Mechanism of the formation of ultrashort pulses in cw solid-state lasers as a result of the optical Stark effect in a saturable semiconductor switch

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1999 Quantum Electron. 29 428

(http://iopscience.iop.org/1063-7818/29/5/A09)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 128.131.79.235
The article was downloaded on 01/07/2010 at 16:43

Please note that terms and conditions apply.
Mechanism of the formation of ultrashort pulses in cw solid-state lasers as a result of the optical Stark effect in a saturable semiconductor switch

V L Kalashnikov, I G Poloi^ko, V P Mikha^ilov

Abstract. It is shown that the Stark shift of a quantum-well level on excitation below the edge of the absorption band in a bleachable absorber may lead to the formation of a pulse of minimum duration. This mechanism also ensures self-starting of the generation of ultrashort pulses.

The rapid progress made in mastering the femtosecond range of ultrashort pulses (USPs), generated by cw solid-state lasers, has recently been associated with a method involving self-focusing in the active element of the laser (see, for example, Rundquist et al. [1] and the references cited there). This method has proved very useful as a procedure for the generation of USPs with durations ranging from several picoseconds to 10 fs. However, its significant disadvantages are the absence of self-starting of the generation of the USPs and the need for careful tuning of the cavity [2]. The laser systems generating USPs with the aid of saturable semiconductor absorbers [3, 4], which make it possible to obtain pulses with a duration of 6.5 fs, are free from these disadvantages [5].

An important feature of such absorber switches is the relaxation of excitation on excitation which is slow (hundreds of femtoseconds), compared with the pulse duration. In order to account for the generation of USPs with durations much shorter than the switch relaxation time, a mechanism was proposed [6] involving soliton stabilisation of a pulse owing to the balance between the phase self-modulation (PSM) and the dispersion of the group velocity. The failure to take into account other, apart from the energy saturation of losses and the associated self-modulation broadening of the absorption band, significantly nonlinear properties of low-dimensional semiconductor structures used as modulators of the laser radiation should be included among the deficiencies of this explanation.

One of the mechanisms of the optical nonlinearity of semiconductor structures may be the Stark shift of an optical resonance of a semiconductor (for example, the exciton resonance in a low-dimensional semiconductor) under the influence of the laser field (see, for example, Knox et al. [7]). As already mentioned by Tsuda et al. [4], the intracavity intensities in real USP-generating solid-state lasers are fully sufficient for the Stark effect to play a notable role. The Stark shift, arising under the influence of an external constant or rf signal, has been used in active mode locking of a diode-pumped Nd:YAG laser [8].

The Stark shift of an optical resonance (the optical Stark effect), which results from the influence of nonresonant transitions on the field-excited transition, may be described within the framework of a generalised quasi-two-level model [9–11]. This shift is proportional to the difference $\Delta z$ between the polarisabilities of the ground and excited states and it may alter significantly the conditions for the propagation of a USP in a nonlinear medium, leading to the formation of an optical soliton [12]. The influence of nonresonant transitions is particularly strong in the interaction of USPs with semiconductor structures, characterised by $|\Delta z| \sim 10^{-19} - 10^{-21}$ cm$^{-1}$ [13]. This yields the Stark shift coefficient $\zeta = 8\pi|\Delta z|/(\omega a) \sim 10^{-3} - 10^{-5}$ cm$^{-1}$ J$^{-1}$ ($a$ is the refractive index).

In the present study, we shall show that the quadratic optical Stark effect induces amplitude self-modulation of the field on excitation at a frequency lower than the optical resonance frequency in a semiconductor. This may lead to the formation of a stable USP of duration much shorter than the relaxation time of the bleaching of a semiconductor structure even in the absence of additional mechanisms of the formation and stabilisation of a lasing pulse.

In the case of weak exciton–exciton and exciton–phonon coupling, we shall postulate a quasi-two-level character of the interaction of a pulse with a low-dimensional semiconductor absorber at a frequency close to the frequency of one of the quantum-well levels. The contribution of nonresonant transitions was taken into account by introducing the difference between the polarisabilities of the unexcited and excited states [9, 10]. The evolution of an off-diagonal density matrix element $\pi$ and of the difference $\eta$ between the populations of the ground and excited states is then satisfied by the equations

$$\frac{d\pi}{dt} + \frac{1}{\tau_e} - i(\omega_a - \omega_e - \Delta \omega) = \frac{2}{\hbar} \eta,$$

$$\frac{d\eta}{dt} + \eta - \eta_0 = - \frac{4}{\hbar} \text{Im}(\pi^e),$$

where $\tau_e$ is the reciprocal of the width of a quantum-well optical resonance at a frequency $\omega_a$; $\omega_a'$ is the instantaneous frequency of the lasing field; $\varepsilon$ is the matrix element representing the interaction with the optical field; $\eta_0$ is the equilibrium difference between the populations; $\Delta \omega$ is the Stark shift owing to the contribution of nonresonant transitions; $T$ is the longitudinal relaxation time.

The evolution of the time profile of the laser field $a(t)$ during a round trip through a cavity containing a semiconductor absorber, an active medium, and a dispersive element is
Mechanism of the formation of ultrashort pulses in cw lasers

described by the operator equation: \( a^{k+1}(t) = \hat{G}(t)A(a; t) \times D(t)a^k(t) \), where \( k \) is the serial number of a pass through the cavity and \( t \) is the local time. The operator \( D(t) = \exp(ik_2G^2/ct^2) \) describes the influence of the group-velocity dispersion (\( k_2 \) is the second-order dispersion coefficient), whereas the operator

\[
\hat{G}(t) = \exp\left( -\frac{gL_g}{1 + L_g^2 G^2 (\dot{c}/\dot{t})} \right)
\]

describes the influence of the active medium with the saturated gain \( g \) and the reciprocal of the width of a Lorentzian gain band profile (we shall consider Cr:forsterite as the active medium with \( t_g = 20 \) fs). The parameter \( L_g \) is given by

\[
L_g = \frac{1}{1 + i(w_{\text{las}} - w_g)G^2},
\]

where \( w_{\text{las}} \) is the carrier frequency of the laser field; \( w_g \) is the centre of the gain band.

In the incoherent approximation [see the system of equations (1)], the action of a saturable absorber is described by the operator

\[
\hat{A} = \exp\left( \left\{-\gamma L_a \exp\left[-\text{Re}(L_a) \int_{t_0}^{t} |a(t')|^2 \right] \times \exp\left( -\frac{t - t'}{T} \right) \frac{dU}{U} \right\} \right) - r,
\]

where \( L_a = \frac{1}{1 + i[w_{\text{las}} - (w_g + \xi |a(t)|^2)]/T} \),

\( \gamma \) represents the losses saturated as a result of filling the upper level at a time \( t_0 \), which corresponds to the USP maximum; \( r \) represents the nonsaturated (residual) losses. The optical Stark effect manifests itself as a field-intensity-dependent virtually instantaneous shift of the resonance frequency, which is described by the term \( \zeta |a(t)|^2 \) in the quasi-monochromatic approximation. The energy saturation of the losses is represented by the integral in the expression for \( \hat{A} \), where \( U_g \) is the saturation density of energy. Here we neglect the contribution of the two-photon absorption to the USP generation process (the validity of this claim will be justified later). For convenience, we shall employ the following normalisation of the dimensional quantities: time is normalised to \( t_g \), the frequency to \( w_g^{-1} \), and the intensity to \( U_g/t_g \). The dimensionless Stark shift coefficient \( \zeta = \zeta U_g \) is 13 for \( U_g = 390 \mu \text{J cm}^{-2} \) reaching a semiconductor switch with PhSe microcrystals dispersed in a glass matrix and having an absorption band which falls, as in the case of PbS [14], within the forsterite laser emission band.

We note that the parameter \( U_g \) may be varied within wide limits (from tens of microjoules to several millijoules per square centimetre) by selecting the excited transition and the switch fabrication technology (the placing of a semiconductor structure in a Fabry–Perot interferometer and its deposition by evaporation onto a Bragg mirror [3, 4] or its fabrication in the form of single crystals dispersed in a glass matrix [14]). For example, in a number of studies [15–18] the measured value of \( U_g \) for CdS/CdSe microcrystals in a glass matrix varied from several microjoules to several millijoules per square centimetre, which clearly indicates the presence of various relaxation channels in the specimens.

Neglecting the second- and higher-order effects in terms of the intensity, we obtain (with the aid of the operator \( \hat{A} \)) the transmission coefficient of a switch, which depends on the instantaneous power of the field reaching it:

\[
M(|a|^2) = \exp\left[ -\frac{-2\Gamma}{1 + (\omega_{\text{las}} - \omega_a)^2 + 2(\omega_{\text{las}} - \omega_a)^2 |a|^2} \right],
\]

where \( \tau = t_0/t_g \); \( \Gamma \) represents the initial losses in the switch associated with the saturated losses \( \gamma \) at the USP maximum in the following manner:

\[
\gamma = \Gamma \exp\left[ -\text{Re}(L_a) \int_{-\infty}^{t_0} |a(t')|^2 dt' \right].
\]

It is seen from formula (2) that, in relation to a field of a frequency less than the resonance frequency, the modulator behaves like a fast saturable absorber. The amplitude self-modulation coefficient is

\[
\rho = \frac{\mu}{M \zeta |a|^2} \frac{\partial M}{\partial |a|^2} |a|^2 \to 0,
\]

where \( \mu \) is the cross section of the lasing mode, equal to 30 \( \mu \)m in our case. The coefficient \( \rho \) is of the order of \( 10^{-7} \) \( \text{W}^{-1} \) (Fig. 1), which is close to the values of this coefficient for systems with mode locking as a result of self-focusing [19]. However, in contrast to the earlier study [19], \( \rho \) depends noncritically on the geometric parameters of the cavity. The physical explanation of the Stark effect as the mechanism of power suppression of losses is that, when the field frequency is less than the resonance frequency, the high-intensity components of the lasing field ‘drive’ the exciton band into the short-wavelength region, diminishing thereby the modulator losses.

In order to investigate the USP characteristics, we shall expand the operator field-evolution equation as a series in terms of the intensities, the energies, and the detunings of the instantaneous pulse frequency relative to the carrier frequency. Neglecting the higher terms, we obtain an equa-

---

**Figure 1.** Amplitude self-modulation coefficient \( \rho \) plotted as a function of the detuning of the laser field from an exciton resonance in the absence (1, 2) and presence of the two-photon absorption (3); \( \Gamma = 0.05 \) (1, 3) and 0.1 (2); \( r = 1, g - r = 0.01; \zeta = 13.\)
tion which is an analogue of the generalised nonlinear Landau–Ginzburg equation (a similar equation was considered by Chen et al. [20]):

\[
\frac{\partial a(k, t)}{\partial k} = \left[ c_1 + i c_2 \frac{\partial}{\partial t} + (c_3 + i c_4) \frac{\partial^2}{\partial t^2} + (c_5 + i c_6) a(k, t) \right]^2 + (c_7 + i c_8) \varepsilon + (c_9 + i c_{10}) \frac{\partial^2}{\partial t^2} + (c_{11} + i c_{12}) \frac{\partial}{\partial t} a(k, t)
\]

where

\[
e = \int_{t_0}^{\infty} |a(k, t')|^2 dt', \quad c_1 = g J_g - \gamma J_a - r;
\]

\[
c_2 = 2g J_g^2 - 2\gamma \omega^2 J_a^2; \quad c_3 = (1 - 3Q^2) J_g^3 - \gamma (1 - 3\omega^2) J_a^3;
\]

\[
c_4 = g(\Omega^2 - 3Q) J_g^3 - \gamma (\omega^2 - 3\omega r) J_a^3 + k_s^2;
\]

\[
c_5 = -2\gamma \omega r J_a^2; \quad c_6 = -\gamma (1 - \omega^2) J_a^2; \quad c_7 = \gamma J_a^2;
\]

\[
c_8 = -\gamma \omega r J_a^2; \quad c_9 = -\gamma J_a^2; \quad c_{10} = \gamma J_g^2;
\]

\[
c_{11} = -\gamma (1 - \omega^2) J_a^2; \quad c_{12} = 2\omega \gamma J_g^3;
\]

\[
J_a = \frac{1}{1 + \omega^2}; \quad J_{\gamma} = \frac{1}{1 + \Omega^2}; \quad H_a = \frac{1}{\left(1 - \omega^2\right)^2 + 4\omega^2 r^2};
\]

\[
\omega = \omega_{\text{las}} - \omega_a; \quad \Omega = \omega_{\text{las}} - \omega_g.
\]

The operation of an instantaneous switch on the basis of the Stark effect is described by a term proportional to the coefficient \(c_1\). Eqn (3) has a soliton-like solution of the type

\[a(k, t) = a_0 \text{sech}^{1/2}(|(t - k\delta)/t_p|)^{1/2} e^{i\omega t},\]

where \(\psi\) represents the chirp; \(\delta\) and \(\varphi\) are the time delay and phase shift of a USP in a round trip through the cavity; \(t_p\) and \(a_0\) are the USP duration and amplitude.

The duration and the detuning from the frequency of the centre of the gain band for a frequency-limited USP as well as the necessary group-velocity dispersion (which may be regulated, for example, by a pair of prisms) are plotted in Fig. 2 as a function of the detuning from the centre of a gain band from an excitation resonance. Evidently, the minimum durations (coinciding approximately with the maximum of \(\rho\) in Fig. 1) are close to the minimum possible duration \(t_g^\ell\) and are in the detuning range \((\omega_a - \omega_g) n_g \approx 0.3 - 0.7\). An increase in the depth of modulation (curves 3 and 1 in Fig. 2a) reduces the pulse duration, which corresponds to an increase in \(\rho\) (curve 2 in Fig. 1). The use of switches with a lower loss-saturation energy increases the pulse duration (curve 2 in Fig. 2a). The anti-Stokes shift of the carrier frequency relative to the gain band increases under these conditions (curve 2 in Fig. 2c). The mechanism of this shift is discussed in Ref. [21].

The excitation resonance band narrowing (increase in \(\tau\)) diminishes the permissible detuning range of the gain band (relative to the absorption band) in which the existence of a USP is possible (curves labelled 4 in Fig. 2). This can be explained by less efficient mode locking owing to an inadequate overlap of the absorber band and the USP spectrum. There is a simultaneous decrease in the phase self-modulation owing to the effect of the Stark shift of the absorption band [the coefficient \(c_8\) in Eqn (3)], which diminishes the negative group-velocity dispersion necessary for chirp compensation (curve 4 in Fig. 2b).

The observed characteristic features of the behaviour of the USP parameters as a function of the detuning from the frequency of the absorption line centre are in qualitative agreement with the experimental data presented by Tsuda et al. [4]. As in the latter study, exact tuning to an excitation resonance (Fig. 2a) does not lead to the generation of a USP. On the other hand, Stokes detuning ensures generation of femtosecond pulses and a subsequent increase in this detuning leads to the generation of picosecond pulses.

An analysis of the stability of the resulting solution, relative to laser noise, is of interest. In order to avoid the appearance of a USP accompanied laser noise, it is necessary to ensure — outside a USP — a negative overall saturated gain, i.e. a negative difference between the saturated (by the energy of a pulse) values of the active-medium gain and of the switch losses when the residual losses are taken into account [22]. Since, owing to saturation of the losses by the pulse energy, the maximum overall gain is reached right at the trailing edge of a USP, we investigated the latter. Fig. 3 presents the overall saturated gain \(\Sigma\) directly behind a pulse for five different noise frequencies \(n_a\). Evidently, the reduction in both the depth of modulation (Figs 3a and 3c) and in the saturation density of energy \(U_a\) (\(U_a = 390\) mJ cm\(^{-2}\) in Fig. 3a and 240 mJ cm\(^{-2}\) in Fig. 3b), and also the narrowing of the absorption band (Fig. 3d) reduce the range of existence of a stable frequency-limited USP. However, stabilisation of a USP may be achieved by the appropriate selection of \(\Omega, U_a\), and \(\tau\).

The destabilisation of a USP with increase in \(\tau\) may be treated as a transition to inefficient mode locking, in particular owing to an inadequate overlap of the spectral contours of the USP and the absorber. The duration of a stable frequency-limited USP is then quite rigidly locked to \(t_g^\ell\). However, for larger values of \(\tau\) there is a possibility of chirped pulse generation (Fig. 4). The dependences of the chirp and of...
the pulse duration on the dispersion then undergo a change in character in the case of small values of \( \tau \) (compare curves 2 and 1 in Fig. 4a). We shall attribute this to the change in the sign of the phase self-modulation, described by the coefficient \( c_0 \), in the case of curve 2. Nevertheless, infinitesimal durations, close to \( t_g \) (Fig. 4b), are also attainable under these conditions. One should note that an increase in \( \tau \) requires allowance for the coherent character of the interaction of a USP with a semiconductor structure, which induces a number of new effects not taken into account within the framework of the present approach. In particular, this may lead to a significant shortening of a USP [12].

Since the most striking experimental manifestation of the effects of the interaction of the laser field with a semiconductor modulator is the self-starting of the generation of a USP, it is of interest to analyse the conditions for such self-starting owing to the optical Stark effect. A criterion of the self-starting is the destabilisation of the steady-state lasing regime (called the self-modulation instability [23]). The solution of Eqn (3), describing steady-state lasing, is of the form \( N = N_0 \exp(i\kappa_0) \), where the amplitude \( N_0 \) and the increment \( \kappa_0 \) may be found from the equations

\[
\frac{g_{\text{max}}^2}{1 + \sigma N_0^2} J_a^2 - \frac{1}{1 + N_0^2} = 2 I \tau_0 N_0^2 \left[ \frac{\sigma N_0^2 J_a^2}{1 - i\alpha T_c} \right] \,.
\]

(4)

The parameters in Eqn (4) have the same significance as in Eqn (2), except that the carrier frequency of a USP must be replaced by the frequency of the steady-state signal; \( \sigma \) is the ratio of the saturation energies of the switch and the active medium (in our case, \( \sigma \approx 10^{-1} \)); the amplitude \( N_0 \) of the steady-state signal is normalised to the loss-saturation power \( \sigma \); \( g_{\text{max}}^2 = 1 \). We shall now investigate the stability of the solution (4) of Eqn (3) in the presence of a complex perturbation of the type \( \nu = \nu_0 \exp(i\kappa) \). The self-starting condition then nominally corresponds to the destabilisation of the steady-state solution as a result of a perturbation with \( \text{Re} \kappa > 0 \) and \( \text{Im} \kappa = \lambda \) (i.e. undamped oscillations with a period equal to that of the cavity). By linearising Eqn (3) with respect to this field perturbation and taking into account the perturbation of the saturated gain, we obtain an expression for the increment \( \lambda \) after adding Eqn (3) to the complex-conjugate equation:

\[
\lambda = g J_a - \gamma J_a + \left( \frac{\varepsilon J_a N_0^2 J_a}{1 - i\alpha T_c} \right) (1 + 9\tau) - \sigma c_2 - \omega^2 c_3 + i\omega^2 c_4 + (c_5 - ic_6) N_0^2 (2 + 9\tau),
\]

(5)

where

\[
\text{Re} \left[ \lambda + \gamma J_a - g J_a + \left( \frac{\varepsilon J_a N_0^2 J_a}{1 - i\alpha T_c} \right) (1 + 9\tau) - \sigma c_2 + \omega^2 c_3 - i\omega^2 c_4 + 2(c_5 - i\omega^2 c_6) N_0^2 \right] \left[ \frac{\varepsilon J_a N_0^2 J_a}{1 - i\alpha T_c} \right] ^{-1}.
\]

The solutions (4) of Eqn (3) in the presence of a complex perturbation may be found from the equations

\[
T_c = \frac{T}{1 + N_0^2 J_a^2} \quad T_c^g = \frac{T^g}{1 + N_0^2 J_a^2} \quad \varepsilon_g = \frac{g_{\text{max}}^2}{(1 + \sigma N_0^2 J_a^2)^2}.
\]
\[ \varepsilon_k = \frac{\Gamma}{(1 + N_0^2 f_k^2)^{\gamma}} \]

\( T^\gamma \) is the relaxation time of the active medium.

The dependence of the imaginary part of the increment in the perturbation of the steady-state solution on the real part is presented in Fig. 5. Evidently, in the three cases corresponding to a stable USP (the parameters correspond to curves 3 in Figs 2 and 3c), steady-state lasing is unstable. However, this does not necessarily imply that USPs are generated spontaneously because in the case of curves 2 and 3 (Fig. 5) there are no perturbation modes at the cavity round-trip frequency. In the case of curve 1, there are two such modes: these are the points of intersection with the \( \text{Im} \approx = 1 \) straight line. For this reason, self-starting of the generation of the USPs as a result of the modulation instability of the steady-state regime is possible precisely for the parameters of curve 1.

**Figure 5.** Perturbation increment for steady-state lasing with \( \chi = 13 \), \( \Gamma = 0.1 \), \( \tau = 1 \), \( g - r = 0.01 \), and \( t_g (\varepsilon_{n_{\perp}} - \varepsilon_{n_{\parallel}}) = 0.7 \) (1), 0.5 (2), and 1 (3).

A factor which needs to be assessed is the contribution of the two-photon absorption, typical coefficients \( \chi \) of which are in the range 2–23 cm GW\(^{-1}\) [24]. Fig. 1 (curve 3) gives the coefficient \( \rho \) obtained taking into account the contribution of the two-photon absorption (\( \chi = 23 \text{ cm GW}^{-1} \)). Evidently, for the parameters analysed in the present study the contribution of the two-photon absorption is small (curve 3 differs only slightly from curve 1).

We have thus shown that the Stark shift of an optical resonance in a quantum-well semiconductor saturable absorber can ensure virtually instantaneous amplitude self-modulation of the lasing field when its frequency is detuned from the resonance frequency. This effect leads to the generation of USPs which are immune to laser noise and have durations close to the reciprocal of the gain-band width. The corresponding nonlinear modulation parameter is sufficiently large to ensure self-starting of the generation of USPs. The principal advantage of the mode-locking mechanism considered here, compared with mode locking as a result of self-focusing, is its insensitivity to a change in the laser cavity parameters.