SELF-MODE-LOCKING OF CONTINUOUS SOLID-STATE LASERS WITH AN EXTRA CAVITY

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As has been recently demonstrated, the mode locking of broadband solid-state lasers using an extra cavity (EC) is a powerful means of obtaining stable trains of ultrashort pulses (USP) of subpico- and femtosecond durations [1-4]. It is based on the possibility for discrimination of fluctuational outbursts of laser radiation due to the interferential interaction of the field from the main cavity with its coherent replica from the EC, which acquired nonlinear phase incursion in the optical fiber. The result of the interference, which depends on the field intensity and phase mismatching of the lengths of the cavities, provides mode-locking of such a system in the course of continuous [4] and synchronous [5] pumping. Recent works have shown that self-mode-locking is possible in a continuous laser and in the case of a linear (empty) EC [6, 7].

The existing qualitative model of self-mode-locking in EC-containing systems does not take into account the fluctuational character of laser radiation and does not allow one to analyze in detail the dynamics of lasing [2, 4]. By contrast, the present work is based on the fluctuational model of generation in continuous solid-state EC-containing lasers [8] which made it possible to reveal some specific features in the self-starting of generation of stable USP trains in such systems.

Simulation involved an analysis of the generation field time profile transformation in passage through the amplifying medium and the spectral filter, with the presence of the EC taken into account. The presence of phase self-modulation (PSM) either in the active medium (AM) or in the EC is supposed. Generation was initiated from spontaneous noise radiation taken into account for the entire lasing time. Permanent noise perturbation made it possible to unambiguously solve the problem of the stability of the resulting stationary solutions, which thus became independent of the initiating random noise sample and were determined by the system parameters as a whole (including the mean statistical characteristics of the noise).

The generation field \( A_k(\eta) \) at the k-th transit was considered on a uniform \( \eta_1, \ldots, \eta_N \) grid of the local time with the interval \( \Delta \eta = \eta_{i+1} - \eta_i \) in the time window equal to the cavity period \( T_{\text{cav}} \). In the presence of PSM in the main cavity, the field was transformed in passage through the AM in conformity with the equation

\[
A_k(\eta) = A_k(\eta) \exp \left[ \alpha(\eta) - iP_a |A_k(\eta)|^2 \right] + S(\eta),
\]

where \( \alpha(\eta) \) is the gain, given by the rate equation [9]

\[
\frac{\partial \alpha}{\partial \eta} = \sigma_{14} (\alpha_m - \alpha) I_p - \sigma_{32} |A_k|^2 - \alpha/T_{31},
\]

\( P_a \) is the PSM coefficient, proportional to the nonlinear refraction factor and to the AM length; \( I_p \) is the pumping intensity; \( \alpha_m \) is the maximum gain with total population inversion; \( \sigma_{14}, \sigma_{32}, T_{31} \) are the absorption and amplification cross sections and the time of the AM excitation relaxation, respectively. The quantity \( S(\eta) \) in Eq. (1) represents the noise source with the Lorentz spectral profile determined by the coherence time \( t_{\text{coh}} \) [10]

\[
S(\eta) = S(\eta - \Delta \eta) \exp \left[ -\Delta \eta/t_{\text{coh}} \right] + S_0 \exp \left[ i\varphi(\eta) \right],
\]

where \( \varphi(\eta) \) is the random phase; \( S_0 \) is the noise level.
The passage of a generation field through a spectral filter in the form of the Fabry–Perot etalon with the transit time \( \Delta \eta \) was described by the equation [11]

\[
A_k(\eta) = (1 - R_f) A_k^0(\eta) + R_f A_k^0(\eta - \Delta \eta),
\]

where \( R_f = \frac{t_f}{(t_f + \Delta \eta)} \); \( t_f \) is the etalon characteristic time, which determines the group lag of the field on the etalon and which is proportional to the filter transmission spectral band width.

The coupling between the field in the main cavity and that in the EC \( B_k(\eta) \) is effected on the exit mirror with the coefficients of reflection \( R \) and transmission \( T \) by means of a splitting plate with the coupling factor \( \theta \) [12]:

\[
A_{k+1}(\eta) = RA^0_k(\eta) - i\theta T \exp(i\varphi) B_k(\eta), \quad B_{k+1}(\eta) = -iT A^0_k(\eta) + \theta^2 R \exp(i\varphi) B_k(\eta),
\]

where \( \varphi \) is the phase mismatching due to interferometric mismatch between the lengths of both cavities.

In the presence of PSM in the EC, its action was described as

\[
B_k(\eta) = B_{k-1}(\eta) \exp(-iP_b|B_{k-1}|^2),
\]

where \( P_b \) is the PSM factor, proportional to the nonlinear refraction factor and to the length of the nonlinear element in the EC, as well as to the ratio between the mode section in the AM and its section on the nonlinear element.

We have selected the following parameters of the system: \( \alpha_m = 1.5, R = 0.78, T_{cav} = 1 \text{ nsec}, \Delta \eta = 1 \text{ psec} \) (i.e., \( N = 1000 \)), \( t_f = 1 \text{ psec}, T_{31} = 3.5 \mu\text{sec} \) (which correspond to \( \text{Al}_2\text{O}_3 : \text{Tl}^{3+} \)). A relatively small period for the cavity was selected for preserving a reasonable time of computations. The coherence time \( t_{coh} \) was selected to be larger than the reciprocal gain bandwidth of the solid-state broadband AM; this means that we considered only a portion of the initial stage of the selection of modes.

It has been found by calculations that with a certain selection of major variable parameters, such as the normalized pumping intensity \( \sigma_{14} T_{cav} I_p \), coupling factor \( \theta \), normalized PSM factors \( P_d/\sigma_{32} T_{cav} \) and \( P_b/\sigma_{32} T_{cav} \) and the phase mismatching of the cavities \( \varphi \), the system generates a stable train of high-intensity USP’s [8]. In Fig. 1 the generation field time profile envelopes are presented at \( k = 1, 50, 500, 1000, \) and 2000. The calculations have shown that the formation of the train passes through three distinct stages. The initial stage (\( k = 1-100 \)) is characterized by rapid exponential growth in the intensity and by mode selection due to the finite bandpass of the frequency filter, which forms large-scale intensity beatings from the primary noise. At the second stage (\( k = 100-1300 \)) the intensity growth rate diminishes because of the removal of inverse population, but the selection of modes persists. The influence of the extra cavity is manifested in the formation of small-scale interference modulation against a background of large-scale intensity beatings. This stage is characterized by the start of the discrimination of pulses due to the PSM-induced growth in the contribution of nonlinear phase incursion resulting in the formation of an ultrashort generation pulse which removes the inverse population; the field outside the maximum pulse is suppressed. The third stage corresponds to the stationary generation of USP with the presence of balance between the gain modulation, inverse population removal, selection of modes and the discrimination of fluctuations due to the interferential interaction. Beyond the generation pulse, one observes only low-intensity noise of the order of the initial noise which initiates lasing. Figure 1 presents the generation field time envelopes which correspond to the linear (\( k = 1, 50 \)), nonlinear (\( k = 500, 1000 \)) and steady-state (\( k = 2000 \)) generation stages. It depicts the trends not only in the self-mode-locking of lasers with an extra cavity, but also in passive mode-locking at large. The occurrence of the USP train generation should not necessarily result from the maximum fluctuation overshoot. It is due to the fact that the dynamics of generation at the local time instant \( \eta \) is determined not only by the value of the fields \( A(\eta) \) and \( B(\eta) \), but also by the value of \( \alpha(\eta) \), which is an integral characteristic of the system, thus adding supplementary field characteristics at different time instants. Thus, even at the initial generation stage the fluctuation overshoots cannot be regarded as statistically independent, and only the latter is a sufficient condition for the maximum fluctuation overshoot in the process of self-mode-locking [10, 13].

Investigation of the dependence of self-mode-locking on the system parameters has shown that generation of stable USP trains is possible only in limited regions of the values of \( \sigma_{14} T_{cav} I_p \) and \( \theta \) with \( P_d/\sigma_{32} T_{cav} \) and \( P_b/\sigma_{32} T_{cav} \) being fixed. The following conditions have been adopted as the criterion of stable self-mode-locking: a) a minimum of 99\% of the whole generation energy over the cavity period should be concentrated in the maximum pulse; b) such concentration should persist for no less than 10\% of the entire generation time, selected by us to be equal to 2000 field transits over the cavity.

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Fig. 1. Time profiles of the logarithm for the normalized generation intensity \( U = \sigma_{52T_{\text{cav}}} \) at 1, 50, 500, 1000, and 2000 transits over the cavity for \( \sigma_{14T_{\text{cav}}}I_p = 0.04, \theta = 0.65, \sigma_{14T_{\text{cav}}} = 0.008, \varphi = 0, t_{\text{coh}} = 1 \text{ psec} \) and other parameters (see text for all the figures in the paper) in the presence of PSM in the main cavity.

Fig. 2. Self-mode-locking regions in the presence of PSM in the main cavity for \( \frac{\sigma}{\sigma_{32T_{\text{cav}}}} = 0.01, \varphi = 0 \) and other parameters and different nature of noise: \( S = \text{const} \) (1), \( t_{\text{coh}} = 10^{-6} \text{ sec} \) (2), \( 10^{-12} \text{ sec} \) (3).

Fig. 3. Dependence of the self-mode-locking development time (1), standard deviation of the peak intensities of USP's (2) and of the delay \( \Delta \) (3) on the pumping intensity for \( \theta = 0.65, \frac{\sigma}{\sigma_{32T_{\text{cav}}}} = 0.008, \varphi = 0, t_{\text{coh}} = 1 \text{ psec} \) and other parameters in the presence of PSM in the main cavity.

Since one of the key features of the model we used is allowance for the fluctuational character of the generation field, it is of interest to consider the influence of noise on the mode-locking regions. We have found that the final results of calculation depend little on the noise level within the range of \( (\sigma_{14T_{\text{cav}}}S_0) \) from \( 10^{-7} \) to \( 10^{-4} \). However, the effect of its correlational properties is much stronger, as seen from Fig. 2. The transition from the case of the stationary initial radiation background with \( S = \text{const} \) to the nonstationary one and the subsequent "aggravation" of the random character of fluctuations with reduction in the coherence time \( t_{\text{coh}} \) (transition from region 1 to region 3 in Fig. 2) diminish the mode-locking region areas and shift them to the side of higher pumpings, thus imposing more stringent demands on the system parameters that ensure USP generation self-starting. This enables us to suppose that the introduction of additional correlations between the values of the field at different time instants can raise the mode-locking efficiency. This can be attained by increasing \( \tau_p \), i.e., by narrowing the transmission band of the frequency-selective element.

The generation dynamics in the interior of the self-mode-locking regions has several special features. The displacement in the regions of Fig. 2 from the self-mode-locking threshold, below which this locking is absent, to the upper boundary, as seen in Fig. 3, reduces the time \( T_0 \) needed for the formation of a generation train. However, as the pumping intensity grows, the mode-locking stability falls sharply, which is characterized by increase in the standard deviation \( \Delta \) of the normal-
Fig. 4. Generation spectrum evolution with an interval of 50 transits up to 1100 transits over the cavity for $P_b/\sigma_{32}T_{cav} = 0.05$, $\theta = 0.7$, $t_{coh} = 1$ psec, $\sigma_{14}T_{cav}I_p = 0.05$ (a, b), 0.4 (c); $\varphi = 0$ (a), -0.2 (b), 0.05 (c) and other parameters in the presence of PSM in the extra cavity.

ized pulse intensities $U = \sigma_{32}T_{cav}I$, with more than 99% of the whole radiation energy over the period $T_{cav}$, throughout the entire generation time considered (Fig. 3, curve 2). The sharp growth in $D$ is associated with the fragmentation of the initially continuous USP train into a number of unsteady trains characterized by the absence of effective mode-locking.

As the pumping intensity grows, the length of the continuous USP chain becomes shorter and on the upper boundary of the region it amounts to less than 10% of the generation time. The USP train breaks because of the appearance of rapidly growing satellites at the generation pulse front.

The preferential additional pumping of the USP front with increasing $I_p$ reduces the time interval between the pulses in the train ($T_{cav} + \Delta$) so that the effective cavity period ($T_{cav} + \Delta$) becomes smaller than ($T_{cav} + t_f$) (Fig. 3, curve 3). If $\delta$ is the time shift of USP under the action of pumping, then $\Delta = t_f - \delta$. Since generation train stability falls with a decrease in $\Delta$ because of amplification of the most nonstationary portion of the generation pulse, i.e., its front, then an attempt at improving the stability by increasing $t_f - \delta$ seems to be reasonable. First, this can be achieved by decreasing $\delta$, i.e., by reducing the pumping level and approaching the self-mode-locking threshold. Second, the stability can be raised by increasing $t_f$, i.e., by narrowing the frequency filter transmission band. Moreover, sharpening of the USP front may have a positive effect when coating losses are introduced into the system or nonstationary noise ahead of the generation pulse is suppressed by superposing a weak field replica onto this section, as is the case when mode-locking is stabilized in synchronous pumping [14, 15]. On the other hand, all the above-enumerated means correspond to the introduction of additional correlations between the fields at different time instants, which, as already noted, also results in increase of the regime stability.

An essential attribute of self-mode-locking of solid-state lasers with an extra cavity and continuous pumping is a strong dependence of the generation dynamics on the phase mismatching of the cavities. Figure 4 presents the evolution of the generation USP spectrum at different values of $\varphi$ for $P_b/\sigma_{32}T_{cav} = 0.05$. From Fig. 4a it is seen that in the case of zero
Fig. 5. Evolution of the generation intensity logarithm profile at the stage of the steady-state generation with an interval of 50 transits from the 100 to the 1200 transit over the cavity for $P_b/\sigma_{32}T_{cav} = 0.05$, $\theta = 0.7$, $\sigma_{14}T_{cav}/p = 0.4$, $\varphi = 0.05$, $t_{coh} = 1$ psec and other parameters in the presence of PSM in the extra cavity.

phase mismatching the original spectrum of Lorentz noise is split into two groups of components symmetric about the carrier frequency $\omega_0$, one of which is thereupon suppressed. To self-mode-locking there corresponds formation of a very broadened spectrum which is displaced into the Stokes region and which has a smooth dome-shaped envelope and virtually a linear "chirp" near the center of the generation pulse.

Introduction of negative phase mismatching $-\pi < \varphi < 0$ (Fig. 4b) does not change the picture substantially, even though the spectrum displacement to the Stokes region becomes smaller. Calculations have shown (see [8]) that the self-mode-locking regions reduce smoothly when $\varphi \rightarrow -\pi$.

Introduction of phase mismatching $0 < \varphi < \pi$ radically changes the generation spectrum character (Fig. 4c). In this case the spectral components are displaced to the opposite (anti-Stokes) region and have a sharply modulated envelope, testifying to the absence of mode-locking. With such mismatching, the self-locking threshold grows sharply and the region diminishes rapidly [8]. In Fig. 5 generation field evolution is presented for positive phase mismatchings. It is clearly seen that at the USP front an extended "pedestal" is formed which incorporates the greater portion of the generation energy. This kind of generation suggests an opportunity for applying coating losses to "filter out the pedestal" and extend the self-mode-locking region in the case of phase mismatching of the cavities.

With PSM present in the main cavity, the foregoing statement about the effect of mismatching $\varphi$ on the generation dynamics remains valid. But in this case both the phase mismatchings and the generation field spectrum displacements are opposite to the case of $P_b \neq 0$ for similar features of the self-locking process.

We note in conclusion that numerical analysis of the fluctuational model for solid-state laser generation having an extra cavity and continuous pumping, with PSM taken into account, made it possible to elucidate the specific features of self-mode-locking in such a system as a function of the nature of noise, pumping intensity and phase mismatching between the cavities. The present analysis allows one to envisage the means for improving the generation characteristics of laser systems with both a nonlinear and linear extra cavity.

REFERENCES