Analysis of the Lasing Dynamics of a Titanium Sapphire Laser with Double Mode Locking and a Limited Train of Ultrafast Pump Pulses

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Titanium sapphire $\text{Al}_2\text{O}_3:\text{Ti}^{3+}$ is among those active media which are presently attracting a lot of attention thanks to their extremely broad amplification band [1-2]. Its ~500 nm wide lasing band and maximum at ~780 nm [2, 3] covers the whole near by IR spectral region allowing high power tunable subpico- and femtosecond pulses to be produced directly from the solid state laser which is significantly useful in many applications. In particular, the use of double mode locking with a short resonator length that is smaller than the length of the pump laser resonator makes it possible to produce a very compact and efficient source of ultrafast pulses with high repetition rates up to 1 GHz, with good beam quality for various spectroscopic applications, including fundamental processes of multiphoton stepped absorption of radiation by materials, high-speed photography, optical computers and communication systems, and electrooptic switches. Increasing the pulse repetition rate and shortening them increases the mechanical and long-term stability of a laser system, as well as improving the reliability of spectroscopic experiments. Achieving the lasing possibilities of titanium sapphire by direct application of known methods for obtaining ultrafast pulses in lasers based on different active media, is prevented, on one hand, by the relatively low amplification cross section $\sigma_{32} \approx 3 \cdot 10^{-19} \text{ cm}^2$ [3] compared with dyes, and, on the other hand, by the short lifetime $T_{31} \approx 3.5 \mu\text{s}$ of the $\text{Ti}^{3+}$ excited state [2-4] compared with activated crystals.

The results of study on the generation of ultrafast pulses from $\text{Al}_2\text{O}_3:\text{Ti}^{3+}$ lasers for various modes of operation and pumping [5-11] stimulated additional methods for achieving the real potential of this medium. This study examines lasing in titanium sapphire with synchronous pumping by a limited train of picosecond pulses and the use of a saturable passive Q-switch (double mode-locking [12, 13]). This transient pumping, from a mode-locked...
solid state laser, for example, is promising as a high intensity pump source for titanium sapphire and is technologically simpler than c-w. The goal of this study is to study the effect pump and Q-switch parameters on the compression of ultrafast pulses.

The investigation was based on analysis of change in the temporal shape of lasing pulses having intensity \( I_k(\tau) \) (where \( k \) is the number of passes, \( \tau \) is the corresponding time) as a result of multiple passes of lasing radiation through the amplifying medium, spectral filtering, saturable absorber, and output mirror which comprise the laser resonator starting at the moment that the first pump pulse is introduced. The equations describing the amplifying field in a four-level \( \text{Al}_2\text{O}_3:Ti^{3+} \) medium with a broad gain band which is significantly broader than the lasing spectral width and a gain band maximum which corresponds to the central frequency of the lasing spectrum, and which take into account pump inversion on the \( 1 \leftrightarrow 4 \) transition due to a pump pulse \( P_k(\tau) \) of duration \( \tau_f \) and spontaneous relaxation of the amplification \( \alpha_k(\tau) \) on the \( 3 \leftrightarrow 1 \) transition with a characteristic time \( \tau_{31} \), were taken from [12]. The effect of the filter which is typically Lorentzian with characteristic time \( \tau_f \) is considered analogous [14] to the time domain. Finally, the absorber is considered to be inertialless with initial absorption \( \kappa \) and saturation intensity \( I_s \).

In the normalized form, the system of coupled equations which correspond to the adopted model, can be written:

\[
\frac{d\alpha_h}{d\tau} = P_h (\alpha_m - \alpha_h) - \alpha_h I_h - \alpha_h/T_{31}, \tag{1}
\]

\[
\left(1 + \frac{\tau_f}{\tau_f + \frac{d}{dt}}\right) I_h = I_h \exp(\alpha_h) + \sigma_{sp} \exp(\alpha_h) - 1, \tag{2}
\]

\[
I_{h+1} = I_h \exp[-\kappa(1 + \sigma I_h) - \gamma], \tag{3}
\]

where the temporal parameters, pump intensity, and lasing pulse intensity are, respectively, normalized to \( \tau_p, (\sigma_{14} \tau_p)^{-1} \) and \( (\sigma_{32} \tau_p)^{-1} \); \( \sigma_{14}, \sigma_{32} \) are the absorption and amplification cross sections of the active media; \( \alpha_m \) is the maximum gain of the medium with complete population inversion of the radiation centers; \( \sigma = (\sigma_{32} \tau_p I_s)^{-1} \) is the saturation parameter of the laser systems; \( \gamma \) is the linear loss in the resonator. For pump and lasing pulses to each transit establish the temporal gain profile from (1), and then the \( I_k \) field from (2), resulting from amplification in the active medium and the effect of the filter, and finally, from (3), the effect of a lasing pulse on the next pass. The inset of lasing at the beginning of pumping \( k = 1 \) is associated with spontaneous emission described by the second term in the right-hand part of (2), where \( \sigma_{sp} \sim T_{31}^{-1} \) is the probability of spontaneous emission [11].

When systems (1)-(3) are solved numerically, the lasing pulse is considered to be distributed over the whole resonator period \( T \), into the left side of which a pump pulse is introduced which has a Gaussian temporal profile and a peak intensity which varies sinusoidally over the pump train by

\[
P_h = P \sin(\pi k/M) \exp(-\tau^2),
\]

where \( M \) is the number of pulses in the train; \( P \) is the maximum peak intensity in the train. The temporal window where ultrafast lasing pulse formation takes place, the largest of which varies from pass to pass, is assumed to be many times larger than the pump duration. Within it, in the region where the lasing pulse varies the fastest, the number of time marks is constant. Outside of this window continuous and negligible variation in the lasing pulse occurs and there are relatively few time marks.

Below, the results are presented of the numerical evaluation for synchronous pumping of an \( \text{Al}_2\text{O}_3:Ti^{3+} \) laser with a limited train of ultrafast pulses for the following parameters of the active medium [2-4], pumping and resonator: \( \alpha_m = 1.5, \sigma_{14} = 1 \cdot 10^{-19} \text{ cm}^2, \sigma_{32} = 3 \cdot 10^{-19} \text{ cm}^2, \sigma_{sp} = 10^{-6}, \tau_p = 50 \text{ psec}, T = 10 \text{ nsec}, \tau_f = 100 \text{ fsec}, \kappa = 0.7, \gamma = 0.5 \). In substituting in (1)-(3), the normalized values of \( P = 1 \) and \( I_k = 1 \) correspond to 80 and 16 GW/cm², respectively. \( \sigma, P, M \) are selected as variables and the mismatch between the oscillator and pump laser resonator periods is \( \Delta T = T - T_p \). The efficiency of generating ultrafast pulses is demonstrated by using pulse characteristics averaged over the pulse train. The average peak intensity \( \langle I \rangle \) and average length \( \langle \tau \rangle \) of the pulse train (arithmetic mean)
are found from the corresponding pulse parameters whose peak intensities differed by less than 90% from the maximum pulse train intensity [15]. Note that the average length in the train can be several times longer than the maximum pulse in the train.

Calculations showed that generation of a train of ultrafast pulses in the systems studied has significant differences from dye lasers, which coupled with the low cross-section of the active medium, requires a comparatively greater pump energy be accumulated for gain modulation. This difference is based on a time lag of greater than 100 nsec between the maximum of the lasing train and the maximum of the pump train which was observed in [7]. As expected, the time lag decreases with an increase in P.
Fig. 5. Average length (solid curves), average intensities of the fundamental lasing pulse peaks (dashed curves) and their secondaries (dot-dashed curves) and their functional dependence on pump mismatch for $P = 0.75$, $M = 30$ and $\sigma = 0$ (1), 1 (2), 100 (3).

Fig. 6. Average length of the lasing pulse train as a function of the saturation parameter for pumping with $P = 0.75$, $M = 30$ and various transient and static losses: $\kappa = 0.1$ (1), 0.3 (2), 0.5 (3, 4), 0.7 (5), 0.4 (6); $\gamma = 0.5$ (1, 6), 0.3 (2), 0.1 (3), 0.9 (4), 0.7 (5).

The average length $<\tau>$ and intensity $<I>$, normalized according to $\tau_p$ and $(\sigma_{32\tau_p})^{-1}$ as functions of the saturation parameter are shown in Figs. 1 and 2 for various pumps $P$ and fixed pulse train $M$. Expressions for curve minima in Fig. 1 show that when setting $\sigma = \sigma_{opt}$ the pulse parameters during lasing are closest to those occurring when efficient pulse shortening by the absorber takes place. The position and depth of the minimum $<\tau>$ depend on pumping: with an increase in $P$ the minimum length decreases, $\sigma_{opt}$ shifts toward lower values corresponding to even sharper switching (greater $I_g$), and the difference between the minimum average length and its value when $\sigma \to 0$ diminishes. Comparison with the case $\sigma \to 0$ is interesting, since it indicates the feasibility of lasing pulse compression by synchronous pumping titanium sapphire lasers with one of two initial loss conditions, in one case part bleaches, in the other, it does not bleach ($\sigma = 0$). With an increase in pumping, the average intensity of the lasing train increases and has a maximum near $\sigma = \sigma_{opt}$ (Fig. 2), corresponding, respectively, best agreement between the rate of growth of gain due to pumping and bleaching of the Q-switch. In the region of this maximum, a distortor in the rounded peaks of the pulse in the lasing train is observed. As seen in Fig. 2, an increase in pumping and the use of still faster Q-switches leads to the onset of the temporal structure of the lasing pulses and an increase in the average peak intensity of the secondary pulses around the fundamental lasing pulses which are shown by the dashed curves. Bleaching of the Q-switch and a decrease in resonator losses earlier after the moment of pumping causes a decrease in the lasing pulses and corresponds to the formation of the secondary pulses which appear at higher pump intensities even in the absence of Q-switch bleaching. The functional dependence of delay time normalized to $\tau_p$ between the fundamental lasing pulse peak at the pulse train maximum and the regular pump pulse peak is shown by the dot-dashed curve in Fig. 2 for a single value of $P$. A negative value of $\sigma_p$ corresponds to a fundamental lasing pulse that leads the pump pulse and excitation of a secondary peak.

The effect of transient pump parameters on the feasibility of minimizing lasing pulse lengths and selection of the optimum absorber for this purpose is shown in Figs. 3-5. As is evident in Fig. 3, the effect of Q-switch on pulse compression is sometimes greater, sometimes less than pumping, i.e., less than the modulation of the active medium due to pumping. With an increase in $P$, the average length of the lasing pulse under "optimum" condition and unbleached Q-switch conditions differ less from each other. This increase in pumping, within
the observed limits, results in a decrease of almost two orders of magnitude in the value of $\sigma_{\text{opt}}$. Uniform average lasing intensities are observed with significantly lower pumping under the condition $\sigma = 0$. This is evidence of the decreased need for synchronous pump intensity for efficient lasing when a bleachable Q-switch is used.

Figure 4 illustrates the effect of pump train length on lasing pulse compression for fixed pump energies $E_p$ and peak intensities $P$ and various saturation parameters. It is obvious that during pumping of the medium at one energy or another, an increase in the number of pulses in the train leads to a flattening of the part which is related to a significant decrease in the value of $<\tau>$ for the case when the "optimum" Q-switch is used and that it more prolonged (compare the solid curves 1 and 2). Note that a twofold decrease in pump energy leads to a value of $\sigma_{\text{opt}}$ that is almost an order of magnitude greater. With a decrease in pump energy and $\sigma = \sigma_{\text{opt}}$ no significant increase in the average length in the corresponding flat part takes place for larger $M$. At the same time, when $\sigma = 0$, the curve for the functional dependence of $<\tau>$ on $M$ is located outside the limits of Fig. 4. An increase in pump energy by increasing the pulse train to $M \approx 100$ a fixed value of $P$, has a significant effect on the rate of growth of gain in titanium sapphire due to the finite relaxation time of the gain. For an unbleached Q-switch, this is determined, on one hand, by the shorter lasing train compared with that in the constant energy case and the smoothing of the curve for the function dependence of $<\tau>$ on $M$ for additional increase in $M$, and on the other hand, by the complicated shape of the lasing pulses expressed by the appearance of one or two secondary pulses resulting from the pump preceding the fundamental pulse. When $\sigma = \sigma_{\text{opt}}$ the Q-switch provides compression of the lasing pulses, which "saturates" with an increase in $M \approx 100$, and filtering out of the secondaries which are exhibited at lower intensities. Thus, use of the "optimum" Q-switch allows, even with a limited pulse train, a train of lasing pulses to be obtained with pulse lengths close to those during continuous synchronous pumping. It is therefore possible to control the value of $\sigma_{\text{opt}}$ over a wide range by varying the pump parameters.

An effective way of obtaining pulses with a bell-shaped temporal profile and none of the secondary pulses that arise from the pump pulse preceding the fundamental lasing peak (Figs. 2, 4), is accomplished by introducing a compensating mismatch $\Delta T$ between the resonator periods of the oscillator and pump lasers. This is seen in Fig. 5, where the average length and intensity of the fundamental lasing pulse peaks and their secondaries are shown as a function of resonator mismatch $\Delta T$, normalized to $\tau_p$ for various saturation parameters. One of the selected values $\sigma = 1$ is close to the "optimum" saturation parameter for $\Delta T = 0$. The minimum lasing pulse length for this is achieved over a wide range of mismatches $\Delta T \leq 0$. For weak Q-switch bleaching (the condition for curves 3), when the laser resonator length is increased relative to the pump resonator, the pumping is introduced into the active medium sooner and the precedence which causes the onset of the lasing pulse secondaries is compensated for. In the limit $\Delta T > 0$ the lasing pulse length becomes minimal. For an un-bleached Q-switch a significantly greater minimum length is manifested with an increase in $\Delta T > 0$.

Still one more possibility exists for minimizing the lasing pulse length by double mode locking coupled with the use of repeated synchronous pumping, where the period of the pump laser resonator is a multiple of the oscillator laser resonator. Curves 4 (Figs. 1 and 2) refer to the condition $T = T_p/2$. It is easy to understand the merits of such pumping which results in an approximately twofold greater lasing train and a greater number of passes of the radiation through the absorber. This is explained by the shorter average pulse length in the train obtained by repeated pumping as well as the maximum peak intensity of the pump train. With repeated synchronous pumping, selection of the "optimum" Q-switch which assures the greatest lasing pulse compression is considerably less critical than the saturation parameter. Over a broad interval of $\sigma$, covering almost two orders of magnitude, $<\tau>$ remains practically unchanged and at a minimum (Fig. 1). At the same time, the lasing pulse has almost the same average fundamental peak intensity over the train as with synchronous pumping for the same value of $P$, but has a simpler temporal shape without secondaries over a broad range of variation in the saturation parameter (Fig. 2).

Figure 6 shows the functional dependence of average train length on $\sigma$ for various transient ($\times$) and static ($\gamma$) laser losses. It is evident that an increase in the ratio of transient losses shifts the minimum lasing pulse compression towards lower $\sigma$, which corresponds to sharper Q-switching. Meanwhile, the minimum length decreases. Furthermore, the
transition to greater total losses for a weak signal \( \kappa + \gamma \) leads to longer lengths for the case of an unbleached Q-switch (\( \sigma = 0 \)) and to shorter lengths in the region around "optimum" saturation.

Note that the value for \( \sigma_{opt} \) obtained, assures the greatest lasing pulse compression with double mode-locking in a titanium sapphire laser, which corresponds to a fast Q-switch and relatively large \( I_s \). Variation of pump and resonator parameters can effect the value of \( \sigma_{opt} \) within broad limits and achieves minimum lasing pulse lengths, close to those conforming to the use of a broadband filter. A significant increase in \( \sigma_{opt} \) is possible due to compression of the lasing band [i.e., an increase in the temporal character of the filter \( \tau_f \) in (2)] [16], while the absolute value of the minimum length achieved with the same pump pulse length increases.

LITERATURE CITED

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