Theory of sub-10 fs generation in Kerr-lens mode-locked solid-state lasers with a coherent semiconductor absorber

V. L. Kalashnikov, I. G. Poloyko
International Laser Center, 65 Skorina Ave., Bldg. 17, Minsk, 220027 Belarus
tel./fax: /375-0172/ 326-286, e-mail: vkal@ilc.unibel.by

ABSTRACT

The results of the study of ultra-short pulse generation in continuous-wave Kerr-lens mode-locked (KLM) solid-state lasers with semiconductor saturable absorbers are presented. The issues of extremely short pulse generation are addressed in the frames of the theory that accounts for the coherent nature of the absorber-pulse interaction. We developed an analytical model that bases on the coupled generalized Landau-Ginzburg laser equation and Bloch equations for coherent absorber. We showed, that in the absence of KLM semiconductor absorber produces $2\pi-$ non-soliton pulses of self-induced transparency, while the KLM provides an extremely short soliton generation. $2\pi-$ and $\pi-$ sech-shaped soliton solutions and variable-squared chirped solutions have been found. It was shown, that the presence of KLM loosens the stability requirements for ultra-short pulse generation removing the limitation on the minimal modulation depth in absorber necessary for the pulse stabilization. An automodulation stability and self-starting ability analysis is presented.

Keywords: ultrashort pulse, semiconductor absorber, self-induced transparency, soliton, Kerr-lens mode locking

1. INTRODUCTION

A rapid progress in ultra-fast lasers has resulted in generation of sub-10 fs pulses, which is close to the fundamental limit defined by the light wave period in near-IR. Today a basic technique for fs-generation is the Kerr-lens mode locking in combination with a slow saturation of interband or excitonic transitions in semiconductor structure. It was shown that in both cases a soliton formation plays the key role. The soliton mechanism stabilizes an ultra-short pulse generation and works efficiently down to shortest possible pulse durations. Moreover, the soliton generation due to its highest stability and simple pulse form is of highest practical importance, which stimulates the investigations in this field.

It was shown in that the strong nonlinear effects in semiconductor absorbers, such as absorption linewidth enhancement and Stark effect, transform the condition for soliton formation, that leads to an additional pulse stabilization and compression. Since the pulse durations in modern ultrafast lasers are comparable or shorter than the absorber dephasing time, the coherent nature of the pulse-absorber interaction has to be taken into consideration. A coherent absorber mode locking has been analyzed in refs. In particular, it was shown in that the dynamical gain saturation is essential for the coherent soliton generation in lasers. However, the latter is negligible in femtosecond solid-state lasers, where the dominating nonlinear factors are self-phase modulation (SPM) and self-focusing. Numerical simulations have demonstrated the generation of fs-pulses of self-induced transparency in solid-state laser with semiconductor quantum-well absorber with no Kerr-lensing in the system. It was shown, however, that Kerr self-focusing is essential in fs time domain and should be taken into account in the analysis. Furthermore, an analytical solution for this problem is still lacking which might provide a new insight into the physics of fs-lasers and semiconductor optical devices.
Here we present a study of fs-soliton generation in cw solid-state Kerr-lens mode-locked laser with semiconductor absorber. We developed an analytical model that accounts for coherent absorption in semiconductor, SPM, KLM, group-velocity dispersion and gain saturation by the full pulse energy. The condition of the soliton (sech-shaped pulse) formation is found and the contribution of Kerr-lensing to soliton formation is demonstrated. We showed that the generation of chirp-free 2π- and π-solitons and chirped quasi-solitons with variable square is possible. The solutions obtained are stable against laser noise with no limitation on the minimal necessary modulation depth introduced by absorber. An automodulational stability and self-starting ability of mode locking are estimated, too.

A rapid progress in ultra-fast lasers has resulted in generation of sub-10 fs pulses, which is close to the fundamental limit defined by the light wave period in near-IR. Today a basic technique for fs-generation is the Kerr-lens mode locking in combination with a slow saturation of interband or excitonic transitions in semiconductor structure. It was shown that in both cases a soliton formation plays the key role. The soliton mechanism stabilizes an ultra-short pulse generation and works efficiently down to shortest possible pulse durations. Moreover, the soliton generation due to its highest stability and simple pulse form is of highest practical importance, which stimulates the investigations in this field.

Fig. 2. Pulse duration $t_p$ versus pump $P$. 2π-pulses ($1, 2$) in the absence and ($1', 2'$) in the presence of SPM. ($2, 2'$) – unstable against the noise solutions. ($3$) 2π-soliton; ($4$) π-soliton; and ($5$) chirped quasi-soliton ($\sigma=0.14, \beta=0.26$) with the KLM present in the system. $\alpha_{max}=0.1$, $T_r=3$ μs, $T_{coh}=10$ ns, $\tau=6.25\times10^{-4}$, $\eta=1$ ($I$, $I'$, $2$, $2'$), 0.5 ($3$), 0.2 ($4$), 0.3 ($5$).

It was shown in refs. that the strong nonlinear effects in semiconductor absorbers, such as absorption linewidth enhancement and Stark effect, transform the condition for soliton formation, that leads to an additional pulse stabilization and compression. Since the pulse durations in modern ultrafast lasers are comparable or shorter than the absorber dephasing time $t_{coh}$, the coherent nature of the pulse-absorber interaction has to be taken into consideration. A coherent absorber mode locking has been analyzed in refs. In particular, it was shown in refs. that the dynamical gain saturation is essential for the coherent soliton generation in lasers. However, the latter is negligible in femtosecond solid-state lasers, where the dominating nonlinear factors are self-phase modulation (SPM) and self-focusing. Numerical simulations have demonstrated the generation of fs-pulses of self-induced transparency in solid-state laser with semiconductor quantum-well absorber with no
Kerr-lensing in the system. It was shown, however\textsuperscript{11}, that Kerr self-focusing is essential in fs time domain and should be taken into account in the analysis. Furthermore, an analytical solution for this problem is still lacking which might provide a new insight into the physics of fs-lasers and semiconductor optical devices.

Here we present a study of fs-soliton generation in cw solid-state Kerr-lens mode-locked laser with semiconductor absorber. We developed an analytical model that accounts for coherent absorption in semiconductor, SPM, KLM, group-velocity dispersion and gain saturation by the full pulse energy. The condition of the soliton (sech-shaped pulse) formation is found and the contribution of Kerr-lensing to soliton formation is demonstrated. We showed that the generation of chirp-free 2$\pi$- and $\pi$- solitons and chirped quasi-solitons with variable square is possible. The solutions obtained are stable against laser noise with no limitation on the minimal necessary modulation depth introduced by absorber. An automodulational stability and self-starting ability of mode locking are estimated, too.

2. MODEL

Based on the slowly varying envelope approximation for the field amplitude $a(t)$, we consider a distributed system including saturable gain $\alpha$, linear loss $\gamma$, SPM $\beta$, Kerr-lens-induced fast saturable absorption with saturation intensity $1/\sigma$, group-velocity dispersion $D$ and bandwidth limiting element with transmission band $1/t_i$. To account for the coherent interaction with the semiconductor absorber while staying in frame of analytic approach we adopted a two-level model of energy levels in semiconductor. This assumption is valid for quantum-confined semiconductor structures utilized in mode-locked fs-lasers.

When the pulse duration is much shorter than the dephasing time in absorber, $t_\tau<<t_{coh}$ and the field intensity $|a(t)|^2$ is not enough for the Stark effect manifestation, the pulse-semiconductor interaction obeys the Bloch equations\textsuperscript{12}:

$$
\frac{du}{dt} = (\Delta - \frac{d\phi}{dt})v + qaw,
\frac{dv}{dt} = -(\Delta - \frac{d\phi}{dt})u,
\frac{dw}{dt} = -qau,
$$

where $u(t)$, $v(t)$ and $w(t)$ are the slowly varying envelopes of the polarization quadrature components and the population difference, respectively, $q=d/\nabla \sigma$, $d=0.28e$ Coulomb$\times$nm is the dipole momentum, $e$ is the elementary charge (in our calculations we used the parameters of GaAs/AlAs absorber, the saturation energy $E_a=0.1$J/cm$^2$ and $t_{coh}=50$ fs), $\Delta$ is the mismatch between optical resonance and pulse carrier frequency, $\phi$ is the instant field phase. An initial saturable absorption $\gamma_0=2\pi N_d^2 \alpha z_{coh} / (c \hbar) = 0.01$ ($\omega$ is the field frequency) corresponds to the carriers density $N=\gamma_0 E_a / (\hbar \alpha z_{coh}) = 2 \times 10^{18}$ cm$^{-3}$ and the thickness of semiconductor absorber $z_{coh}=10$ nm.

The laser part of the master equation is the generalized Landau-Ginzburg equation\textsuperscript{2}. Then the master laser equation can be written as:

$$
\frac{\partial a(z,t)}{\partial z} = \left[ \alpha - \gamma + i \theta + \delta \frac{\partial}{\partial t} + (r_f^2 + iD) \frac{\partial^2}{\partial t^2} + \frac{\sigma - i \beta}{\eta^2} |a|^2 \right] a + \frac{2\pi N_a \alpha d}{c} u - \frac{2\pi N_d \alpha d}{c} \frac{\partial v}{\partial t},
$$

![Fig. 3. Minimal modulation depth of absorber $\gamma_0$ for (1) stable and (2) unstable 2$\pi$-pulse, (dashed curve) generation threshold and (dotted curve) maximal modulation depth for $\pi$-pulse. Dashed region is a stability zone. $\eta=1$ (1), 0.2 (dotted curve).](http://spiedigitallibrary.org/ on 04/12/2013 Terms of Use: http://spiedl.org/terms)
where $z$ is the longitudinal coordinate normalized to the cavity length, i.e. the number of the cavity round-trip, $c$ is the light velocity, $\theta$ and $\delta$ are the phase and time field delay after the cavity round-trip, respectively. We neglected the spatial effects in the absorber. In what follows we shall consider only steady-state pulsed solutions, which allows to eliminate the dependence on $z$. We normalized the times to $t_f$ and the field to $q_{tf}$, $\beta$ and $\sigma$ are normalized to $(2qt_f^2/nce_0 = 5 \times 10^{12} \text{ cm}^2/\text{W}$, where $n$ is the index of refractivity, $\varepsilon_0$ is the permittivity and $t_f=2.5$ fs (Ti: sapphire laser). With this normalizations $\sigma=0.14$ corresponds to the saturation parameter of Kerr-lens induced fast saturable absorber of $10^7$ W and 30 $\mu$m spot size in active medium. Dimensionless SPM parameter $\beta$ is equal to 0.26 for 1 mm thick Ti: sapphire crystal. (Note, however, a

dimensioned pulse duration in Figs. 2 and 4). Additionally, we introduced an important control parameter $\eta$, which is governed by 1) the ratio between the size of generation mode in active medium and in semiconductor absorber or by 2) the reflectivity of the upper surface of the semiconductor saturable device. Formally, the variation of $\eta$ means the variation of relative contribution of SPM, Kerr-lens-induced saturable absorption or saturable gain with respect to the saturable absorption in semiconductor.

As we shall see later, the gain saturation by the full pulse energy is an important factor for the pulse stability and thus should be taken into account. The simplest way to do this is to use a quasi-two level model for active medium. After some calculations for the gain saturated by the full pulse energy $E$ in the steady-state condition we have

\begin{align*}
\sigma &= 0.02, 0.04, 0.06, 0.08, 0.10 \\
\eta &= 0.2, 0.4, 0.6, 0.8, 1.0 \\
\tau_f, \tau_s &= 20, 15, 10, 5, 0
\end{align*}

Fig. 4. Pulse duration $\tau_p$ versus (solid and dashed curves) $\eta$ and (dotted curve) $\sigma$ in the presence of KLM for (1) $2\pi$-soliton; (2) $\pi$-soliton; (3) chirped quasi-soliton for $\sigma=0.14$, $\beta=0.26$; (4) chirped quasi-soliton for $\eta=0.2$, $\beta=0.26$. $P=0.001$ for all curves.
\[
\alpha = \frac{P\alpha_{\text{max}}}{P + \sigma_{\text{id}}T_{\text{coh}}/h\nu_p}, \quad \text{where } \alpha_{\text{max}} \text{ is the gain for the full population inversion, } T_r \text{ is the gain relaxation time normalized to the cavity period } T_{\text{cav}}, \sigma_{\text{id}} \text{ is the dimensionless pump intensity, } \nu_p \text{ is the pump frequency, } \sigma_{\text{id}} \text{ is the absorption cross section of active medium, } I_p \text{ is the pump intensity, } \tau = 6.25 \times 10^{-4} \text{ is the normalized inverse energy of the gain saturation.}
\]

Later we will consider the different simplifying realizations of our model aimed to investigation of the pulse solution of the system (1, 2).

3. PULSE OF THE SELF-INDUCED TRANSPARENCY IN THE ABSENCE OF KLM

We consider the case \( \Delta = 0 \) and restrict ourselves to the case of chirp-free solutions. After integration the equations (1) the master equation (2) reads:

\[
\frac{\partial a(z,t)}{\partial z} = \left[ \alpha - \gamma + i\theta + \delta \frac{\partial}{\partial t} + (1 + iD) \frac{\partial^2}{\partial t^2} + \frac{\sigma - i\beta}{\eta^2} |a|^2 \right] a - \frac{\gamma a}{t_{\text{coh}}} \sin(\psi(z,t)),
\]

where \( \psi(z,t) = \int_0^t a(z,t')dt' \) is the pulse square (note, that the field and time are dimensionless quantities here). Under steady-state condition (the pulse envelope is independent on \( z \)), an integro-differential Eq. (3) transforms to the differential equation

\[
\frac{d^2 a(x)}{dx^2} + \frac{\gamma a}{t_{\text{coh}}} \sin(x) = 0.
\]

In the absence of the lasing factors we have a well-known nonlinear equation of the Klein-Gordon's type with \( 2\pi \)-soliton solution in the form \( a(t) = a_0 \text{sech}(t/t_o) \), where \( a_0 \) is the amplitude, \( t_o \) is the duration\(^2\). But this solution does not satisfy the full Eq. (4) in the absence of KLM (\( \sigma = 0 \)). Further we consider the Eq. (4) neglecting the KLM, SPM and GVD (\( \sigma = \beta = D = \theta = 0 \)).

The substitution \( \psi(t) = x, \frac{d\psi(t)}{dt} = y(x) \) (“square-amplitude” representation) reduces the third-order Eq. (4) to the second-order one:

\[
\frac{d^2 y}{dx^2} + \left[ \frac{\gamma a}{t_{\text{coh}}} \sin(x) \right] = 0.
\]

We solved this equation numerically and found a nonsoliton (i.e. non-sech-shaped) \( 2\pi \)-solutions for it (see Fig. 1, where numerical solution is shown in comparison with the sech-shaped pulse). This clearly indicates on some additional nonlinear factors which are necessary for the soliton formation and which are absent in Eq. (5).

However, \( 2\pi \)- nonsoliton solution is worth more detailed investigation for it might be relevant mechanism for ultimately short pulse generation in real lasers. As is known\(^1\), the main mechanism of destabilization of fs-pulses is the noise generation as result of loss saturation in absorber. In the case of \( 2\pi \)-pulse formation, the Rabi flopping of the absorber population suppresses the noise behind the pulse tail, thus stabilizing fs-generation\(^10\).

To investigate the pulsed solutions of Eq. (5) analytically, we used a harmonic approximation: \( y(x) = a_1 \sin(x/2) + a_2 \sin(x) + \ldots \). Retaining only the first term, in the “square-amplitude” representation, we arrive to the solution \( a_1 = 2\sqrt{2(\alpha - \gamma)}, \quad \delta = \gamma a/(2(\alpha - \gamma)T_{\text{coh}}), \quad t_o = 2a_1, \) which corresponds to sech-shaped solution in the “time-amplitude” representation. The relations between pulse parameters are analogous to that ones for \( 2\pi \) sech-shaped solution, except an additional relation arising between pulse amplitude and lasing factors \( \alpha \) and \( \gamma \).

Fig. 2 (curves 1 and 2) presents the pulse durations for two physical solutions of Eq. (5). One can see, that the coherent absorber provides sub-10 fs pulse generation starting from some minimal pump. An important feature of this solution is the positive difference between saturated gain and linear loss \( \alpha - \gamma > 0 \), which imposes a requirement on the possible minimal saturable loss \( \gamma_s \) necessary for stable pulse generation, i.e. the minimal modulation depth of absorber that confines the region of the pulse stability against laser noise. As it was shown in\(^r\), the pulse is stable if the net-gain outside pulse is negative that produces the condition \( \alpha - \gamma - \gamma_s < 0, \) i.e. \( \gamma_s > \alpha - \gamma \). The dependence of the minimal modulation depth on the pump is shown in Fig. 3 for two physical solutions of Eq. (5) presented in Fig. 2 (lower curve corresponds to the solution with larger duration, the dashed curve depicts the generation threshold, hatching shows a corresponding stability zone). As one can see, the pulse stabilization against laser noise is possible only for the solution with longer duration. The stability range widens in \( \gamma_s \), however at the cost of pump growth, which is the obvious disadvantage of this regime.
Now we introduce into equations the SPM and the GVD terms, that corresponds to a real femtosecond lasers. Assuming a chirp-free nature of possible solution we can reduce Eq. (4) to the first-order equation making use of a pure real character of the field:

\[
\delta \frac{dy(x)}{dx} + \frac{\beta}{\eta^2 D} y(x)^2 + (\alpha - \gamma - \frac{\theta}{D}) y(x) - \frac{\gamma a}{t_{coh}} \sin x = 0.
\]

(6)

With the above described harmonic approximation we have the following solution:

\[
\theta = \frac{3\beta a_1^2}{4\eta^2 + D(\alpha - \gamma)}, \quad \delta = 4\gamma a/(t_{coh} a_1^2), \quad a_1 = 2\sqrt{3(\alpha - \gamma)}, \quad t_p = 2/a_1 \quad \text{again for } 2\pi\text{-squared solution.}
\]

The pulse duration is presented in Fig. 2 by curves 1' and 2'. The stable against the noise solution has a slightly longer duration than the unstable one.

Formally, the solution obtained is the solutions of the laser part of master equation that in the same time satisfies the Bloch equations. The stability of the solution results from the self-induced transparency in absorber, when the pulse propagates in the conditions of the positive net-gain and noise suppression due to Rabi flopping of the absorber population. We do not analyze the automodulational stability\textsuperscript{13} of the solution since it has been testified directly by the numerical simulation in\textsuperscript{10}.

Thus we can conclude, that there is not the generation of coherent soliton in the absence of Kerr-lens-induced fast saturable absorption. But the generation of nonsoliton 2π-pulse takes place, which imposes a limitation on minimal modulation depth of the absorber with subsequent growth of generation threshold. In the next section we take into account the contribution of KLM.

4. COHERENT 2\pi-SOLITON IN THE PRESENCE OF KLM

The presence of KLM is described by the term \( \sigma a_1^2 \) in Eq. (3). In this case there is a 2π-soliton solution of Eq. (3), however for a strict relation between \( \sigma \) and \( \eta \), so that \( \sigma = \frac{\eta^2}{2} \). The sech-shaped solution has the following parameters:

\[
a_0 = \frac{2}{t_p}, \quad t_p = \frac{1}{\sqrt{\gamma - \alpha}}, \quad \delta = \frac{\gamma a}{t_{coh} (\gamma - \alpha)}
\]

(7)
There are two distinct features of $2\pi$-soliton generation: 1) the condition $\alpha-\gamma < 0$ is satisfied and, consequently, there is no limitation on the minimal modulation depth of the absorber; 2) the expression for the pulse duration is precisely the same as for the case of pure fast saturable absorber mode locking, that suggests that the KLM is the main mechanism determining the pulse duration. This conclusion corroborates with the results of ref.11. The action of the coherent absorber determines the pulse delay $\delta$ and imposes a restriction on the pulse square, i.e. the relation between pulse duration and amplitude. Pulse duration in the presence of KLM is shown in Fig. 2 by dotted curve 3. As is seen, the pulse duration is much shorter than for the case with no KLM, especially for the small pump. As the contribution of semiconductor absorber is increased ($\eta$ approaches 1, curve 1 in Fig. 4) the pulse duration is reduced down to the limit of the validity of the slowly varying envelope approximation.

An explanation of additional relation between the parameters of Kerr-lens-induced absorber and semiconductor absorber is as follows: a sech-shaped solution satisfies both pure laser equation and the Bloch equations, but the coherent interaction discriminates the special cases of $n\pi$–squared pulses, in particular $2\pi$–pulses, which are provided by $\sigma=\eta^2/2$ relation.

Let us study an automodulational stability of the laser coherent soliton, which we showed before to be very important factor in fs-lasers. We used an aberrationless approximation, which assumes an unchanged form of solution and $z$-dependence for the pulse parameters. The substitution of the pulse envelope in Eq. (3) with following expansion into the time series yields:

$$\frac{da_0}{dz} = 2\frac{(\alpha-\gamma)\eta^2 t_p^2 - \eta^2 + 4\sigma}{\eta^2 t_p^2}, \quad \frac{dt_p}{dz} = \frac{4\eta^2 - 2\sigma}{a_0^2 \eta^2 t_p^2},$$

$$\theta = \frac{2(D + 4 \frac{\beta}{\eta^2})}{a_0^3 t_p^3}, \quad \delta = \frac{2}{t_{coh} a_0^3}.$$

Eqs. (8) were derived for a chirp-free solution with the dispersion $D=-2\beta/\eta^2$ exactly compensating for SPM. To be self-consistent the system (8) should be completed by additional relation between pulse duration and amplitude arising from the Bloch equations. After some calculations we have the explicit expressions, which determine the pulse stability. The pulse is stable if the Jacobean of the right-hand sides of first two Eqs. (8) has only non-positive eigenvalues. The condition for amplitude perturbation decay $-4(\gamma-\alpha)^2 < 0$ is satisfied automatically. For $\sigma=\eta^2/2$ the pulse possesses a marginal stability with respect to pulse duration. However for any $\sigma<\eta^2/2$ the pulse stability condition with respect to duration is satisfied.

Thus, we have analyzed the characteristics of $2\pi$-solitons generated in fs KLM lasers with semiconductor coherent absorber. However, equations describing this physical situation allow yet another type of solutions.

5. COHERENT $\pi$-SOLITON AND CHIRPED QUASI-SOLITONS IN THE PRESENCE OF KLM

As one can see from the previous part of our work, the ultrashort pulse is the soliton for the both laser part of the master equation and the Bloch equations. Now we consider the complex ansatz describing a pulse with chirp $\zeta$:

$$a(t) = a_0 \text{sech}(\frac{t}{t_p})^{1-i\zeta}.$$

It is known, that the Eqs. (1) have a sech-shaped solutions in form of chirp-free $\pi$-pulse or sech-shaped chirped solution, when the following relations hold:

$$u(t) = u_0 \text{sech}(\frac{t}{t_p}), v(t) = v_0 \text{sech}(\frac{t}{t_p}), w(t) = \tanh(\frac{t}{t_p}), \frac{d\phi(t)}{dt} = \frac{\zeta}{t_p^2} \tanh(\frac{t}{t_p}),$$

where

$$a_0 = \frac{1}{\sqrt{1+\zeta^2}}, u_0 = -\frac{1}{\sqrt{1+\zeta^2}}, v_0 = \frac{\zeta}{\sqrt{1+\zeta^2}}.$$

A chirp-free solution is a $\pi$-soliton, which is obviously unstable in the absorber since the full population inversion behind the pulse tail amplifies the noise. However, another nonlinear factors in KLM-laser can stabilize the pulse and this requires a corresponding consideration.

Parameters of the chirp-free $\pi$-soliton in KLM-laser are:

$$a_0 = \frac{1}{t_p}, t_p = \frac{1}{\sqrt{\gamma-\alpha}}, D = -\frac{\beta}{2\eta^2}, \theta = \frac{\beta(\gamma-\alpha)}{2\eta^2}, \sigma = 2\eta^2.$$
This is very similar to $2\pi$-solution (7), however with some differences. As the pulse amplitude of the $\pi$-soliton is two times smaller than for the $2\pi$-soliton, a KLM-parameter in order to produce the same effect and support soliton generation should be increased four times and the dispersion should be decreased accordingly.

Curve 4 in Fig. 2 depicts the pulse duration for $\pi$-soliton. As is seen, the pulse duration slightly differs from the duration for $2\pi$-soliton (curve 2) and is shorter in the region of small $\eta$ (curve 2 in Fig. 4).

As was said before, a $\pi$-soliton inverts the population difference in the absorber that causes the noise amplification behind the pulse tail. Hence, the laser pulse stabilization is possible if the condition $a+2\sigma<0$ is satisfied. This defines the maximal initial loss in the absorber (dotted curve in Fig. 3). It is seen, that the maximal modulation depth exceeds the threshold (dashed curve) and, consequently, the generation of the stable $\pi$-soliton is possible.

When the condition

$$\eta^2 < \frac{3\sqrt{\sigma^2 + \beta^2} - \sigma}{4}$$

holds there are the physical chirped solutions with sech-shape. In this case, the expressions for $D$ and pulse parameters are bulky and we do not write them here. The pulse duration is presented in Fig. 2 by curve 5. As is seen, the chirped pulse duration can be very short even for the moderate value of $P$. There is a minimum in the dependence of the pulse duration on $\eta$ (an optimal reflectivity of semiconductor absorber device, curve 3 in Fig. 4) and it does not coincide with the point of precise chirp compensation (curve 1 in Fig. 5, a). Additionally, the pulse square is variable for this type of solution (curve 1 in Fig. 5, b).

There is a sharp minimum in the dependence of the pulse duration on $\sigma$, also (curve 4 in Fig. 4). For our parameters a corresponding KLM-parameter is $7\times 10^7$ W. As it was in the previous case, the minimum of the pulse duration does not coincide with the point of chirp compensation (curve 2 in Fig. 5, a). Unlike the case of variation of $\eta$, the variation of $\sigma$ causes only a slight variation of the pulse square (curve 2 in Fig. 5, b).

Summarizing, the generation of sech-shaped $\pi$-pulses and chirped pulses with variable square is possible in KLM-lasers with coherent semiconductor absorber as a result of definite relation between KLM and saturable absorber’s contribution. A larger KLM contribution (for a fixed $\eta$) is needed to produce the pulse as compare to the case of $2\pi$-soliton.

### 6. SELF-STARTING ABILITY

Our results suggest that the pulse duration is determined rather by KLM, whereas a saturable absorber puts a limitation on the pulse square. But an important feature of KLM in the presence of semiconductor absorber is the self-starting ability. To estimate it in our model we analyzed an evolution of the initial noise pulse, which is much longer than the relaxation time of the excitation in absorber $T_r=1$ ps. For such a noise pulse an absorber is fast and the action of SPM and self-focusing is negligible. Using the normalization of the time, gain saturation energy and field intensity to $T_c$, $E_a$ and $E_0T_c$, respectively an evolution equation for the noise pulse is:
where all notations have the meaning as before, and field parameters refer to the noise pulse. To solve Eq. (10) we used, as before, an aberrationless approximation. The decay of the noise pulse (growth of the duration and decrease of the intensity) means in our model that the system will not self-start. An opposite situation with an asymptotic growth of the noise pulse testifies about ability of the system to self-start. After some calculations, the self-starting conditions are:

\[
\begin{align*}
\frac{d^2 a(z,t)}{dt^2} + (1 + 4T_a a_0^2 + 4T_a^2 a_0^2) - \gamma_a T_a a_0^2 t_p^2 > 0, \\
-\gamma_a T_a a_0^2 (PT, \eta^2 + 2T_a T_0 T_0 + 2a_0^2 + 2a_0^2 T_0 T_0 + \eta^2 + 2a_0^2 T_0 \eta^2 T_0) + \\
PT, \eta^2 (\alpha_{\text{max}}^2 - 2\alpha_{\text{max}} t_p^2 T_0 - t_r^2 - 2T_a T_0 - 2T_a^2 T_0) - 2T_a T_0 \tau_p (1 + 2T_a^2) \\
- \gamma_a T_a a_0^2 (PT, \eta^2 + 2a_0^2 T_0 \eta^2 T_0 + \eta^2) > 0.
\end{align*}
\]

Fig. 6 demonstrates the regions of the initial pulse parameters corresponding to the self-starting. The dark zone corresponds to the pump \( P = 8.5 \times 10^4 \), which is close to the threshold of mode locking self-start. Lower pump cannot provide the self-starting while a higher pump (\( P = 8.8 \times 10^4 \)) causes an expansion of self-starting region.

7. CONCLUSION

In conclusion, we investigated the conditions of soliton formation in cw solid-state laser with coherent semiconductor absorber. It was found, that the mode locking in the absence of Kerr-lens-induced fast saturable absorption does not produce sech-shaped pulse. This \( 2\pi \)-pulse (pulse of self-induced transparency) has fs-duration and is stabilized by the defined minimal modulation depth of absorber. The stabilization results from the positive difference between saturated gain and linear loss and an increase of the threshold of sub-10 fs generation. A combined action of KLM and coherent absorption produces the sech-shaped soliton, which removes a requirement to the minimal modulation depth of semiconductor absorber. The pulse duration, which is close to the fundamental limit, is defined by KLM and the coherent absorber defines the pulse square. As result, there are \( 2\pi, \pi \)-solitons and chirped quasi-soliton with variable square.

Our results can be useful for the development of high-efficient self-starting generators of extremely short pulses for fs spectroscopy, X-ray and THz generation.

All calculations in this paper were carried out in Maple V, the corresponding commented programs are presented at http://www.geocities.com/optomaplev.

8. REFERENCES