Basle Accord vs. Value-at-Risk Regulation in Banking *

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Regulation in Banking

ABSTRACT

During the last couple of years regulatory capital requirements have suffered from a gap between economic risk and risk as it was seen by regulators. This paper compares regulation based on the Basle Accord to Value-at-Risk or “internal model” based capital requirements. For this purpose we develop a continuous time framework that allows banks to switch between two different risk levels. This leads to a proper understanding of the risk taking (switching) incentives created by these two regulatory mechanisms. The main finding is, that Value-at-Risk based capital regulation is – in a world with two risk levels – pareto superior in terms of higher equity values and lower deposit insurance liability when compared to traditional capital adequacy regulation.

JEL classification: G21; G28; G12
I. Introduction

[W]e have no choice but to continue to plan for a successor to the simple risk-weighting approach to capital requirements embodied within the current regulatory standard. While it is unclear at present exactly what that successor might be, it seems clear that adding more and more layers of arbitrary regulation would be counter productive. We should, rather, look for ways to harness market tools and market-like incentives wherever possible, by using banks’ own policies, behaviors, and technologies in improving the supervisory process.

Greenspan (1998)

Especially during periods of financial crises as in 1999 in Asia prudential bank regulation is paid a lot of attention from researches, government agencies, and industry professionals. While there is still an ongoing debate, whether regulation is beneficial at all\(^1\), we can see that regulation is an evolving process and a large number of regulatory guidelines have been issued by the Basle Committee on Banking Supervision and by national regulators during the last couple of years.

One of the milestones of banking regulation is the 1988 Basle Accord (Basle Committee on Banking Supervision (1988)), where regulators set up minimum capital requirements that are calculated by classifying assets into different categories, that should link minimum capital to credit risk\(^2\). The idea was to require banks to hold capital as a safety cushion that should be able to absorb losses. The level of capital should be linked to the risk of the banks assets, thus penalizing banks for taking excessive risk, as it was e.g., observed during the savings and loans crisis in the 80ies, when authorities hesitated so close insolvent banks, inducing them to increase risk as bank managers hoped to gain solvency again. This approach, however, was subject to criticism by academics and practitioners since capital requirements were somehow

\(^1\)see e.g. Freixas and Rochet (1989), pp. 257 for a survey
\(^2\)Later the same idea was adopted for market risk out of trading activities (Basle Committee on Banking Supervision (1997))
related but not linked close enough to the economic risk\textsuperscript{3}. Borrowers are grouped into categories and capital requirements are homogenous within each of these buckets defined in the Basle Accord, whereas the market risk premium is directly linked to the borrower’s individual credit risk and therefore different within a bucket.

Due to increased market pressure large financial institutions developed their own risk management models, not only to control the bank’s exposure to market risk but to optimize capital allocation within the firm\textsuperscript{4}. Since optimal capital holdings computed by these internal models are substantially different from regulatory capital requirements, regulation causes serious distortions in the optimal portfolio choice of banks. Banks take advantage of this problem by “regulatory capital arbitrage” (see Jones (2000)) that is minimizing equity requirements within the bank, by holding those assets where the difference between market risk premium and regulatory capital is most favorable and selling others. This leads to a higher risk per regulatory capital ratio and therefore also to a higher return on capital.

Regulators are aware of this problem and are concerned about economic costs due to mis-allocation and of banks’ distorted incentives towards risk taking, and are interested in a reform of the Basle Accord. Today in most countries banks are allowed to choose between computing minimum capital requirements according to their own internal risk management models or use the Basle method for their trading books. If banks want to use internal models, regulators require banks to have enough equity capital to cover a maximum loss – the Value-at-Risk, that will not be exceeded with a certain probability. A lot of – especially large banks – have chosen to compute capital requirements using their internal models. In the future this approach may be extended to the banking book as well.

This paper studies risk taking incentives as well as welfare effects under Basle Accord and Value-at-Risk regulation. In a first stage we analyze the bank’s optimal risk taking policy for

\textsuperscript{3}e.g. while there is no capital required to back a loan to the OECD member state South Korea, loans to AAA rated firms have to be backed by full 8% equity.

\textsuperscript{4}see e.g. Froot and Stein (1998) or Stoughton and Zechner (2000)
a given regulatory mechanism. Then we compare the liability of the deposit insurance fund and the value of equity under the two regulatory regimes.

The analysis is set up in a continuous time framework allowing banks to choose between two different levels of risk. Since we explicitly model the process of risk shifting we are able to derive the dynamic choice of asset risk. Compared to Basle Accord regulation we find that Value-at-Risk based capital requirements provide a strong incentive for the solvent bank to reduce asset risk. This advantageous characteristic of Value-at-Risk regulation benefits directly the regulator by reducing the deposit insurance liability. Furthermore, when properly applied solvent banks will also vote in favor of Value-at-Risk regulation because lower capital requirements for low risk banks benefit the bank’s equity holders. From a social planer perspective Value-at-Risk regulation can be pareto superior.

There are two branches of literature related to our approach. The first – addressing the issue of bank regulation in a continuous time framework – has been founded by Merton (1977). He derives the insurance premium of a fixed length deposit contract applying the Black and Scholes (1973) option pricing framework. Merton (1978) introduces random audits by the regulator and derives the fair up-front price of deposit insurance under the assumption of a constant volatility of the bank’s assets. Pennacchi (1987) considers risk taking incentives by banks, where he defines risk in the form of financial leverage. He also points out the importance of regulatory response to a bank failure in the sense that he compares direct payments to depositors to merging a failed bank. Fries, Mella-Barral, and Perraudin (1997) consider optimal bank closure rules balancing social bankruptcy costs against future auditing costs. They also find incentives for managers to take risk, where risk is defined as the volatility of the underlying state variables and not as leverage, and they derive subsidy policies and equity support schemes that eliminate these risk taking incentives by linearizing the equity holders’ value function. Finally Bhattacharya, Plank, Strobl, and Zechner (2000) derive optimal closure rules, that eliminate risk taking incentives for managers – at least in the region, where the bank would not be closed by regulators. All these models above assume that the volatility of
the underlying state variable is constant. The existence of a risk taking incentive is deduced solely from the convexity of the value function, however, the process of risk shifting is not explicitly treated.

The second branch of literature examines risk shifting in a continuous time corporate finance setting. Ericsson (1997) and Leland (1998) introduce models, where equity holders are allowed to switch from one risk level to another. They value the different claims on the firm and derive an optimal capital structure policy. We extend these models and use them in a banking context to derive the results stated above.

The paper is composed as follows: Section II describes the model, Section III derives the general solution of claims on the banks assets, Section IV compares Basle Accord and Value-at-Risk regulation and explores the risk taking incentives created by these mechanisms. Section V derives comparative statics, undertakes welfare considerations and gives some policy implications for prudent regulation and in Section VI we conclude.

II. Model

As in Merton (1974), the value of the banks assets $V$ is assumed to follow the process

$$\frac{dV}{V_0} = \frac{\mu - \delta}{\mu} V dt + \sigma V dz,$$

$$V(0) = V_0 > 0$$

(1)

where $\mu$ and $\delta$ denote the total expected return on asset value $V$ and the constant fraction of asset value paid out to the security holders respectively. The differential $dz$ is the increment of a standard Wiener process and the instantaneous variance of the process $V$ is $\sigma^2 V^2$. However, we allow the management of the bank, whose interests are assumed to be perfectly aligned.

5All models so far assume, that management acts in the interest of equity holders. A recent contribution by John, Saunders, and Senbet (2000) examined the interesting idea of linking bank regulation to management compensation and thereby inducing bank owners to write contracts with bank managers, that induce them to pursue an optimal investment policy.
with the equityholders’

6 to choose \( \sigma \), the risk of the bank’s assets to be either high or low, that is \( \sigma \in \{ \sigma_L, \sigma_H \} \). Thus, the state of a bank is characterized by the location in the two-dimensional state space \([0, \infty) \times \{ \sigma_L, \sigma_H \}\) spanned by the range of \( V \) and \( \sigma \). Switching the risk of the portfolio is not costless. Managers have to pay proportional switching costs of \( kV \), whenever they shift portfolio risk from one level to the other. This assumption seems troublesome in today’s markets, where almost arbitrary risky positions can be obtained in the derivatives market at very low trading costs. However, these positions are very obvious to regulators, since they have to be reported periodically and there would not be a intense audit necessary to unveil them. We interpret risk here as a more subtle risk of a loan portfolio, where shifting the risk structure goes in hand with notable trading costs. The regulatory agency has to perform an audit in order to get a detailed status of the portfolio’s risk.

We assume that the bank has issued deposits with face value \( c/r \) (where \( r \) is the riskless rate of interest) which requires a continuous coupon flow \( c \). These deposits are fully insured by the regulator, which means that in case of bankruptcy the depositors receive the full face value. Equityholders have limited liability and are the residual claimholders of the bank’s assets. If – on closure – the asset value \( V \) is not sufficiently high to cover the claim of the depositors, the difference is borne by the regulatory authority.

Coupon payments are assumed to be tax deductible, i.e., as long as the bank serves its obligations it receives a tax benefit of the magnitude \( \tau_c \). In times of distress equityholders are free to inject money to guarantee solvency in order to keep the prospect of future benefits from the tax shield or – alternatively – they may voluntarily close the bank.

Despite voluntary closure there exists the possibility of closure forced by the regulatory authorities if the state of the bank is not in accordance with the regulatory mechanism implemented. In this article we consider regulatory mechanisms \( \langle \lambda, B(\sigma) \rangle \) characterized by (i) an auditing intensity \( \lambda \) and (ii) by a minimum capital requirement \( B(\sigma) \). In detail:

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6In common with the previous literature equityholders are assumed to be risk neutral.
• Audits are assumed to occur randomly following a Poisson process with intensity \( \lambda \). That means, we model an audit-counter \( A \) defined by the stochastic differential equation

\[
\begin{align*}
dA &= \begin{cases} 
1 & \text{with probability } \lambda dt, \\
0 & \text{with probability } (1-\lambda) dt,
\end{cases} \\
A(0) &= 0,
\end{align*}
\]

which is incremented by one at any occurrence of an audit.

• The minimum capital requirement \( B(\sigma) \) defines the consequences of an audit by partitioning the state space of the bank into a ‘closure region’ \( (V < B(\sigma)) \) and a ‘continuation region’ \( (V \geq B(\sigma)) \). Due to the fact that our model allows for two levels of asset risk \( (\sigma_L \text{ and } \sigma_H) \) the bank perceives only the two critical thresholds \( B(\sigma_L) \) and \( B(\sigma_H) \) from the entire minimum capital requirement that is implemented.

For a given regulatory mechanism \( (\lambda, B(\sigma)) \), bank management sets an optimal response in order to maximize equity value. At any point of the state space the available choices are (i) stick to the current risk level, (ii) switch the level of asset risk, and (iii) close the bank. In particular, a strategy \( S \) of the management is a mapping from the state space into the space of available choices,

\[ S : (V, \sigma) \rightarrow \{\text{stick, switch, close}\}. \]

In technical terms, switching and closure points are absorbing barriers to the asset value process. While the first hit of a closure point results in the default of the bank the first hit of a switching point \( (\hat{V}, \sigma_H) \) absorbs the high volatility process and creates a low volatility process at \( (\hat{V}, \sigma_L) \). Analogously, a switching point at \( (\hat{V}, \sigma_L) \) absorbs the low volatility process and creates one with high volatility at \( (\hat{V}, \sigma_H) \).\footnote{We do not allow for two opposite switching points \( (\hat{V}, \sigma_H) \) and \( (\hat{V}, \sigma_L) \) – i.e., with identical asset value but at different risk levels. This guarantees that the asset value process is properly defined.} The commitment to stick to the current risk level
is – from the point of view of our model – the decision to stay passive and observe the dynamic evolution of the asset value.

Obviously, the possible structure of such a strategy might be very complex. However, an entirely disordered set of sticking, switching and closing points cannot be the optimal response to a regulatory mechanism with simple structure as assumed above. As it is the management’s task to find the optimal switching and closure thresholds we study the class of strategies $S$ where switching points and closure points are bounds of intervals with constant volatility (or in other words, for given volatility the partition of the state space where $S = \text{stick}$ is the union of open intervals). Inside these intervals of stable volatility the asset value $V$ follows a simple geometric Brownian motion (see (1)). Consequently, given a strategy $S$ the value of any claim contingent on the bank’s asset value can be obtained by standard contingent claims analysis when proper boundary conditions are applied at the respective switching and closure points (see Section III).

Concluding this section we will summarize the different claims contingent on the state of the bank $(V, \sigma)$ that will be used to analyze the model and give their characteristics.

- The market value of deposits – denoted as $D(V, \sigma)$ – is the market value of the uninsured coupon flow provided by the bank. In contrast to the insured contract held by depositors which is worth $c/r$, the claim $D$ is exposed to default risk.

- The value of the deposit insurance – denoted as $DI(V, \sigma)$. This is the current value of possible future expenditures necessary to guarantee the full face value to depositors in case of bank closure. They are borne by the regulatory authority. Obviously, the value of the deposit insurance is the difference of the insured value of deposits and the market value of the coupon flow, thus,

$$DI(V, \sigma) = \frac{c}{r} - D(V, \sigma).$$ (3)
• The tax benefits – denoted by $TB(V, \sigma)$ – which are the current value of the profit flow originating from the tax shield $\tau c$.

• The switching costs – denoted by $SC(V, \sigma)$ – which are the current value of future costs from shifting the portfolio risk from $\sigma_L$ to $\sigma_H$ or vice versa.

• The value of equity – denoted by $E(V, \sigma)$ – which is simply the residual value

$$E(V, \sigma) = V + TB(V, \sigma) - D(V, \sigma) - SC(V, \sigma).$$ (4)

III. Valuing a Claim Contingent on ($V, \sigma$)

The issue of this section is the valuation of a claim contingent on the state of the bank ($V, \sigma$). The valuation equations will be derived investigating a general claim $F(V, \sigma)$ which covers all the characteristic features of the claims involved in our model. The adaptation of the general results to the special claims $D, TB$, and $SC$ is relegated to the Appendix, DI and $E$ can be obtained using (3) and (4).

Suppose $F(V)$ is a claim contingent on $V$ and – for a given $\sigma$ – the thresholds $V_1$ and $V_2$ ($V_1 < V_2$) are bounds of stable regime (see Section II). That means there are (i) no switch points and (ii) no closure points inside these bounds and (iii) the interval $(V_1, V_2)$ either belongs entirely to the ‘closure’ region ($V_2 \leq B(\sigma)$) or it entirely is in the ‘continuation’ region ($B(\sigma) \leq V_1$). Furthermore, this claim provides (iv) a constant profit flow $\alpha$ as long as the process $V$ is inside these bounds, (v) on closure at some $\hat{V}$, the claim pays $\beta + \gamma \hat{V}$.

If agents are risk neutral – demanding for a constant interest rate $r$ – market equilibrium requires that $F$ satisfies the second order ordinary differential equation

$$rF = \frac{1}{2} \sigma^2 V^2 F_{VV} + (r - \delta)VF_V + \alpha + I_{[0,B(\sigma)]}(\beta + \gamma V - F)$$ (5)
inside the interval \((V_1, V_2)\), where \(I_{[0,B(\sigma)]}\) denotes the indicator function over the interval \([0, B(\sigma)]\) and \(F_V, F_{VV}\) are the first and second partial derivatives of the claim value with respect to \(V\).

The general solution of this equation is – in the case that \(V\) is in the closure region – given by

\[
F(V, \sigma) = \frac{\alpha}{r+\lambda} + \lambda \left( \frac{\beta}{r+\lambda} + \frac{\gamma}{\lambda+\delta} V \right) + A_1 V_{x_1(\sigma)} + A_2 V_{x_2(\sigma)}. \tag{6}
\]

Outside this region the solution is

\[
F(V, \sigma) = \frac{\alpha}{r} + A_1 V_{y_1(\sigma)} + A_2 V_{y_2(\sigma)}. \tag{7}
\]

The constants \(x_1(\sigma), x_2(\sigma), y_1(\sigma), y_2(\sigma)\) are the positive and the negative root of the characteristic quadratic polynomial of the respective homogeneous differential equation

\[
\frac{1}{2} \sigma^2 x(\sigma) [x(\sigma) - 1] + [r - \delta] x(\sigma) - [r + \lambda],
\]

\[
\frac{1}{2} \sigma^2 y(\sigma) [y(\sigma) - 1] + [r - \delta] y(\sigma) - r. \tag{8}
\]

Thus, inside an interval of stable regime the value of the claim \(F\) is entirely characterized by (6) and (7) respectively which are the analytical solution of the Hamilton-Jacobi-Bellman equation (5). The only unknowns remaining are the two parameters \(A_1\) and \(A_2\) which must be determined by boundary conditions at the bounds of this interval, hence, for simplicity and as an abbreviation for Equations (6) and (7) we write

\[
F = F(V, \sigma; A_1, A_2). \tag{9}
\]

In our model the canonical bounds which determine intervals of stability are

1. switching thresholds,
2. closure thresholds set by the bank’s management,

3. the bounds $B(\sigma_L)$ and $B(\sigma_H)$ of the closure region resulting from the mechanism $(\lambda, B(\sigma))$, and

4. the critical value $c_r$; at this threshold the functional form of the default payoff of the deposit insurance contract changes – below $c_r$ the default payoff is $V - \frac{c}{r} < 0$, since the deposit insurance has to bear the difference between the asset value and the face value of deposits. Above $c_r$ the default payoff to the deposit insurance claim is zero, since bank’s assets sufficiently cover deposits.

A. Boundary Conditions

We finish the theoretical part deriving the boundary conditions that have to be satisfied at these four types of boundaries by the different kinds of claims involved in our model. The analysis in this section is, again, done with the general claim $F$, the explicit expressions for equity, deposits, tax benefits and switching costs are relegated to the Appendix. After that we draw our attention on an example.

1. Suppose $\sigma_c \in \{\sigma_L, \sigma_H\}$ denotes the volatility at the current risk level and $\sigma_{-c}$ is the volatility at the alternative risk level. Furthermore, let $V_i$ be a switching threshold set by the bank’s management at which the assets are reorganized to a portfolio with volatility $\sigma_{-c}$. Let $F(V, \sigma_c; A_1, A_2)$ denote the market value of the claim prior to the volatility shift at $V_i$ and $F(V, \sigma_{-c}; A'_1, A'_2)$ the claim value subsequent the volatility shift in the neighborhood of $V_i$ (according to the convention (9)). Furthermore assume that in the case of a switch expenditures of $kV$ are required ($k \neq 0$ only for the claim SC which denotes the current value of all future switching costs, and via (4) for the equity value $E$).
Market equilibrium requires

\[ \lim_{V \to V_i} F(V, \sigma; A_1, A_2) = F(V_i, \sigma; A'_1, A'_2) - kV_i, \]  

(10)

Where the limes is the left hand side or the right hand side limes, depending whether \( V_i \) is the upper or the lower bound of the interval of stable regime. This results in an equation which is linear in the four unknowns \( A_1, A_2, A'_1, A'_2 \) and allows to eliminate one of these parameters.

2. Suppose \( V_i \) is a trigger at which the bank’s management decides to default, i.e., \( V_i \) is an absorbing barrier to the process \( V \). Again, depending on the state \((V, \sigma)\) of the bank the market value of the claim prior to default can be written as \( F(V, \sigma; A_1, A_2) \). Since the claim pays \( \beta + \gamma V \) in case of closure, market equilibrium requires

\[ \lim_{V \to V_i} F(V, \sigma; A_1, A_2) = \beta + \gamma V, \]  

(11)

which eliminates one of the unknown parameters \( A_1, A_2 \). Again, the limes is either the right hand side or the left hand side limes, depending on the position of the closure point relative to the interval of stable regime.

3. Suppose \( V_i \) is the bound of the closure region corresponding to the current asset volatility \( \sigma \), i.e., \( V_i = B(\sigma) \). In contrast to the boundaries discussed in the previous two points \( V_i \) now is not an absorbing barrier, but the process \( V \) can freely enter and leave the closure region. Thus, according to the results of Feynman and Kac (see Björk (1998) or on a more formal level Karatzas and Shreve (1988)) market equilibrium requires that the value function of the claim is continuous and smooth at the boundary of the closure region:

\[ \lim_{V \to V_i^-} F(V, \sigma; A_1, A_2) = \lim_{V \to V_i^+} F(V, \sigma; A'_1, A'_2), \]

\[ \lim_{V \to V_i^-} F_V(V, \sigma; A_1, A_2) = \lim_{V \to V_i^+} F_V(V, \sigma; A'_1, A'_2). \]  

(12)
This condition yields two equations linear in $A_1, A_2, A_1', A_2'$ which eliminate two of these parameters.

4. Suppose $V_i = c/r$ and the functional form of the claim’s default payoff changes at $c/r$. Again, $V_i$ is not an absorbing barrier, thus, boundary condition (12) has to be satisfied at $c/r$. Note, the functional form changes at $c/r$ only for deposits and via (3) and (4) for deposit insurance and equity value respectively. For tax benefits and switching costs condition (12) leads to $A_1 = A_1'$ and $A_2 = A_2'$.

5. The last case we need boundary conditions for the situation where the interval of stable regime is unbounded – either from above or from below. Let $F(V, \sigma; A_1, A_2)$ denote the market value of the claim and, first, suppose $V_2 = \infty$, i.e., the interval of stability is unbounded from above. A growing asset value $V$ leads to the fact, that a switch of the regime of stability in the foreseeable future gets less likely. Thus, for growing $V$ the market value of the claim has to converge to the market value the profit flow has if the current regime prevails forever. Excluding speculative bubbles we receive the boundary condition

$$\lim_{V \to \infty} F(V, \sigma; A_1, A_2) = \begin{cases} \frac{\alpha}{r + \lambda} + \lambda \left( \frac{\beta}{r + \lambda} + \frac{V}{\lambda + \delta} \right) & : B(\sigma) = \infty \\ \frac{\alpha}{r} & : B(\sigma) \neq \infty \end{cases}$$

(13)

Second, suppose $V_1 = 0$, i.e., the interval of stability is unbounded from below. Regarding that $V = 0$ is a fixed point of the process (1) it is easy to determine the market value of the claim at $V = 0$. Market equilibrium requires that

$$\lim_{V \to 0} F(V, \sigma; A_1, A_2) = \begin{cases} \frac{\alpha}{r + \lambda} + \lambda \left( \frac{\beta}{r + \lambda} \right) & : B(\sigma) \neq 0 \\ \frac{\alpha}{r} & : B(\sigma) = 0 \end{cases}$$

(14)

In both cases the respective boundary condition eliminates one of the unknowns $A_1$ and $A_2$. 

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B. Optimality Conditions

The management’s issue is to find the operational strategy which maximizes the equity value. As it is stated in Section II the instruments are the choice of switching points and the commitment of exit thresholds which have to be fixed simultaneously. The first order conditions for switching and closure points that are bounds of intervals of stability imply smoothness at the respective boundaries (see Dixit (1993) for a discussion of the so called ’smooth pasting conditions’). Derivation of boundary conditions (10) and (11) leads to:

- if \((\hat{V}, \sigma_c)\) is an optimal switching point

\[
\lim_{V \to \hat{V}} E_V(V, \sigma_c) = E_V(\hat{V}, \sigma_{-c}) - k. \tag{15}
\]

- if \((\hat{V}, \sigma_c)\) is an optimal closure point

\[
E_V(\hat{V}, \sigma_c) = 0, \tag{16}
\]

IV. BA versus VaR – Comparison of two Regulatory Regimes

Based on the framework developed in the last two sections we now consider two regulatory regimes – Basle Accord (BA) and Value-at-Risk (VaR) regulation. We discuss the differences in capital requirements and analyze the implications of these regulatory mechanisms on the risk taking behavior of the bank management.

A. Capital Requirements

One of the main ideas of the 1988 Basle Accord was to require banks to back all their assets with 8% of equity capital. For asset classes that were considered less risky by regulators, like loans to the government and supranational organizations, OECD banks and asset backed
residential mortgage loans, there are discounts to this capital requirement.\textsuperscript{8} Without loss of
generality, we assume, that the asset mix of the bank among these classes is constant and
risk shifting will only occur within these classes\textsuperscript{9}. The bank in this model has a very simple
balance structure. The assets with current market value $V$ are on the asset side. The liabilities
are represented by perpetual deposits with a constant instantaneous coupon of $c$ and face value
$c/r$, where $r$ denotes the riskless interest rate. The regulator’s goal is to preserve a safety
cushion, such that the value of the assets $V$ is sufficient to satisfy the depositors’ claims $c/r$.
Under BA regulation, the minimum cushion $V - c/r$ is determined by the risk weighted assets
of the bank. Depending on the bank’s borrowers this capital requirement will be a fraction $\rho$
of the bank’s assets\textsuperscript{10}. In the case of an audit, the bank will be allowed to continue operation
only if

$$V - \frac{c}{r} \geq \rho V$$

(17)

According to our model framework, the minimum capital requirement $B(\sigma)$ under the BA
mechanism is constant for all possible volatilities of assets in the same group, i.e.,

$$B(\sigma)^{BA} = \frac{c}{r(1 - \rho)}.$$  

(18)

VaR regulation demands that – in case of an audit – banks 99\% Value-at-Risk for a ten
days time horizon must be covered by $V - \frac{c}{r}$, the difference between asset value and the face
value of debt.

Since the asset value of the bank $V$ follows a geometric Brownian motion (see Equation
(1)), the returns are normally distributed with mean $(\mu - \delta - \frac{1}{2}\sigma^2)T$ and an standard deviation
of $\sigma\sqrt{T}$. The factor $T$ scales the moments of the distribution to a horizon of ten days, e.g., if $\mu$,

\textsuperscript{8}See e.g. Dewatripont and Tirole (1994) for a more detailed description and analysis of the Basle Accord.
\textsuperscript{9}This allows to examine the effect of regulatory capital arbitrage, see Jones (2000)
\textsuperscript{10}If the bank’s portfolio only consists of loans to individuals and corporations $\rho$ would be 8\%, if the loan
portfolio only consists of asset backed mortgage loans it would be 4\% etc.
δ, and σ are measured with respect to the time unit of one year (= 250 trading days), we have $T = 10/250$. After linearizing and neglecting the mean of the distribution, as it is done in most VaR implementations, the 99% quantile of the loss distribution is given by $\Phi^{-1}(0.99)\sigma\sqrt{T}$, where $\Phi^{-1}(0.99)$ is the 99% quantile of the standard normal distribution. Or in other words, the bank is allowed to continue its operation if

$$V - \frac{c}{r} \geq a\sigma V,$$  \hspace{1cm} (19)

where $a = \Phi^{-1}(0.99)\sqrt{T} \approx 2.33\sqrt{T}$. The minimum capital requirement for VaR regulation is, thus, given by

$$B(\sigma)^{VaR} = \frac{c}{r(1-a\sigma)},$$  \hspace{1cm} (20)

Comparing equations (18) and (20) we can see that the main difference between the two regulatory regimes is, that VaR regulation explicitly accounts for the risk of the portfolio by adjusting the capital requirements, while the BA regulation is independent of the volatility of the institution’s assets. This has great implications on the risk taking (shifting) incentives the management is exposed to which will be discussed in the following subsection.

**B. Risk Shifting Incentives**

The risk taking incentive which leads bank managers to increase an institution’s risk stems from the fact that the deposit insurance corporation has given a put option on the bank’s assets to the equityholders.\textsuperscript{11} This put option’s value increases with the volatility of the underlying and, thus, makes higher risk more favorable to equityholders. The reason is that high volatility creates an upside chance while the downside risk is bounded by limited liability. Since deposits are fully insured, risk taking is at the expense of the insurance corporation. To mitigate

\textsuperscript{11}The strike of this option according to our model assumptions is $(1-\tau)\xi$, which is the current value of tax adjusted coupon payments.
this problem, different regulatory responses have been discussed in the literature all of them focusing on resolving the convexity in the equityholder’s value function. Fries, Mella-Barral, and Perraudin (1997) suggest state dependent subsidies and equity support schemes to make the equity function linear for troubled banks, Bhattacharya, Plank, Strobl, and Zechner (2000) choose the closure threshold and the auditing intensity such that the value function is linear for solvent banks (i.e., for banks whose asset value satisfies the minimum capital requirement). We consider Basle Accord and Value-at-Risk regulation which are regulatory mechanisms of the form \( \langle \lambda, B(\sigma) \rangle \), consisting of an audit intensity \( \lambda \) and a minimum capital requirement \( B(\sigma) \) (see Section II). If one of the randomly occurring audits reveals an asset value below the capital requirement, the bank is forced to close. Hence, equityholders are not entirely free in setting the optimal closure point for the bank with the consequence that they cannot fully exploit the benefit of the put option. Depending on \( \lambda \) and \( B(\sigma) \) the risk taking incentive is weakened or managers might even find it beneficial to reduce asset risk.

Under BA regulation the minimum capital requirement depends only on a coarse classification of the bank’s assets and not on market risk. Therefore, the auditor’s toughness (i.e., choosing a high \( \lambda \)) is the only way to mitigate risk taking. Figure 1 illustrates the impact of different audit intensities on the equity value by means of an example. When \( \lambda \) is low, the convex shape of the simple put option prevails over the entire range of the underlying. Whereas under strict auditing, the curvature of the equity value changes its sign. When the asset value is significantly below the capital requirement, an audit will result in the immediate closure of the bank. Since higher asset volatility increases the chance that the bank recovers before the next audit takes place it is preferred to low volatility. Or in other words, if the bank is really in distress, the management has a strong incentive to gamble for resurrection – regardless of the audit intensity. When the capital requirement is met, there is still the positive effect of high volatility on the equity value that stems from limited liability on closure. However, high volatility increases the probability that the bank runs into distress and that it will – due auditing – be closed inefficiently early (at least from the equityholders’ point of view). If \( \lambda \) is sufficiently high the negative effect dominates the positive and the solvent bank prefers low risk.
Figure 1. Bank equity value $E$ under Basle Accord regulation as a function of the asset value $V$ for high and low audit intensities plotted against the asymptote. The vertical line represents the minimum capital requirement. While for low audit intensities convexity prevails, high audit intensities create an incentive to reduce risk for the solvent bank.

to high risk. Solvent banks reducing their assets’ risk essentially reduce the deposit insurance corporation’s liability, thus, this is – from the regulators point of view – a desirable behavior. Nevertheless, whether the bank managers really switch the risk level and when they optimally do it, also depends on the costs for rearranging the portfolio and can only be answered after analyzing the model with a particular parameterization (see Section V).

VaR regulation enhances the incentive for solvent banks to reduce risk by setting the minimum capital requirement according to the actual asset risk. Since higher asset volatility implies higher capital requirements (see Equation (20)) a bank can improve its solvency by reducing the asset risk. Suppose, the asset value is between the capital requirement for low risk $B(\sigma_L)$ and the requirement for high risk $B(\sigma_H)$. If an audit occurs and the bank is running low risk the audit confirms solvency (i.e., no negative consequences for the bank). If – in the same
situation – the bank’s portfolio consists of high risk assets an audit results in bank closure. This fact creates a great incentive to reduce asset risk. Looking at the solvent bank we see that under VaR regulation the distance to the respective closure threshold $B(\sigma)$ is larger for the low risk portfolio than for the high risk portfolio, enhancing the reduction in the probability of future distress when switching to low volatility.

According to the motivation above we consider the following risk taking strategies: In principal the region where a bank sticks to its current risk level ($S(V, \sigma_c) = \text{stick}$) is unbounded from above and bounded from below by a closure point. However, the incentives for the low risk bank to increase risk when it runs into distress may justify a switch to high risk before the closure point is reached. On the other hand, the solvent high risk bank may find it profitable to reduce risk at a finite threshold. Whether switching is optimal or not has to be determined by comparing equity values. The locations of switching and closure points are determined
Figure 3. Bank equity value $E$ under VaR regulation as a function of the asset value $V$. The vertical lines represent the two different closure thresholds. The two functions show the equity value for high risk ($\sigma = \sigma_H$) and low risk ($\sigma = \sigma_L$) respectively. While the bank prefers high risk when it is insolvent it reduces risk when sufficiently solvency is regained.

by the optimality conditions given in Section III.B. Figure 2 illustrates the case when the auditing frequency is sufficiently high to induce a shift towards low risk. The switching costs (which are a dead weight loss) are responsible for the fact that the bank’s possible states form a hysteresis. When the solvent bank’s asset value deteriorates, the management will switch to high risk at a certain threshold $S_H$. If the asset value drops further equityholders will close the bank voluntarily at the level $B^*$. Recovering from distress, bank’s management will wait until $S_L$ to switch back to low risk. Between these switching thresholds there is a non-unique correspondence between asset value and asset risk.

Figure 3 shows the corresponding equity value as a function of the asset value. Despite the convexity of the high risk value function the VaR regulation (together with the appropriate $\lambda$) gives enough incentive to the solvent bank to switch back to low risk.
Table I
Parameter values for the numerical analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>coupon of debt</td>
<td>c</td>
</tr>
<tr>
<td>riskless interest rate</td>
<td>r</td>
</tr>
<tr>
<td>face value of debt</td>
<td>c/r</td>
</tr>
<tr>
<td>corporate tax rate</td>
<td>τ</td>
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<tr>
<td>audit frequency</td>
<td>λ</td>
</tr>
<tr>
<td>return-volatility of low risk technology</td>
<td>σₗ</td>
</tr>
<tr>
<td>return-volatility of high risk technology</td>
<td>σ₉</td>
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<tr>
<td>switching costs</td>
<td>k</td>
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<tr>
<td>dividend payout rate</td>
<td>δ</td>
</tr>
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<td>ρ</td>
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<tr>
<td>Value-at-Risk confidence level</td>
<td>p</td>
</tr>
<tr>
<td>Value-at-Risk holding period</td>
<td>T</td>
</tr>
</tbody>
</table>

V. Results and Comparative Statics

In this section we present the potential benefits of VaR compared to BA regulation by means of a numerical example. For this purpose we first have a closer look at the mechanics behind the optimal risk choice and derive some comparative static results. Second, we analyze these regulatory mechanisms from a welfare perspective. Unless otherwise stated, we take the parameter values from Table I12. Due to the higher risk sensitivity of the VaR mechanism the VaR regulated bank switches between the two possible risk levels. BA regulated banks find it optimal to stick with high risk.

In Figure 4 the location of the critical thresholds is plotted against the auditing frequency λ. The minimum capital requirements $B(σₗ)$ and $B(σ₉)$ are not affected by the auditing policy of the regulator. The regime switching points show intuitive behavior. Equityholders

12Note that the “panic factor” is not included in calculating Value-at-Risk capital requirements. As outlined in Basle Committee on Banking Supervision (1996) this multiplicator is “designed to account for potential weaknesses in the modeling process” such as fat tails in the distribution of risk factor returns, sudden changes in volatilities and correlations, intra day trading, event risk and model risk (especially with options). Generally, applying a panic factor will strengthen the risk reduction incentives because it makes capital requirements more sensitive to risk.
Figure 4. Location of the critical thresholds for different values of audit frequency $\lambda$. While regulatory capital requirements are not affected by the auditing intensity, a tougher regulator makes it less attractive for shareholders to support an ailing bank. The high risk technology also becomes less desirable, managers switch earlier back to low risk as the audit intensity increases.

are less willing to support an insolvent bank as the probability of an audit increases. The critical asset value $B^*$ at which the equity holders will close the bank therefore increases with the audit frequency. The switching point $S_H$, where equityholders switch to high risk and start to gamble is determined by two offsetting effects. If the value of the banks assets is below the closure threshold $B(\sigma_L)$, a higher probability of an audit leaves the equityholders with less time to bring the asset value up again. A more stringent auditing policy therefore puts additional pressure on management to start gambling. But, once switched to higher risk, capital requirements increase. With high risk banks have to have higher capital requirements and these are harder to meet before the next audit. This effect together with dead weight
switching costs\textsuperscript{13} prohibits early switching. In this example the two effects offset each other. A similar trade-off determines the location of the point $S_L$, where the manager switches from high risk back to low risk. The switch reduces the value of the deposit insurance option, resulting in a value gain from switching late. On the other hand switching allows the manager to be more relaxed, since the closure region has then moved further away plus the probability of getting into trouble has decreased. As the regulator becomes tougher ($\lambda$ increases), the latter effect dominates and managers have an incentive to switch earlier. If the auditor reduces $\lambda$ below a certain level $\lambda_{\text{min}}$ (which is $\approx 0.37$ in our example) the threat of an audit is not taken seriously by banks and instead of switching back to low risk they will focus on exploiting the deposit insurance option.

The impact of a higher volatility of the risky technology is explored in Figure 5. An increase in the volatility $\sigma_H$ of the “high risk technology” lowers the point, at which the equityholders voluntarily decide to close the bank. This is because a higher volatility gives them a greater value of gambling for resurrection by increasing the probability of bringing the asset value above the closure threshold again. This effect is not compensated by the increase of the capital requirement $B(\sigma_H)$, which is approximately linear for small changes in the financial institution’s risk. The increased attractiveness of gambling also makes it more advantageous for low-risk banks to start gambling by switching to high risk once the asset value has gone below $B(\sigma_L)$. Finally, the switching point $S_L$, where high risk banks switch to low risk again, is substantially increasing with $\sigma_H$. As equity is like a call option on the bank’s assets, the option value is increasing with volatility. This effect dominates the gain from reduced insolvency risk when switching to low risk. Beyond a certain level (which is $\approx 0.24$ in our example) equityholders see no reason to switch back to low risk.

\textsuperscript{13}Since switching costs are assumed to be proportional to the asset value there is a general incentive to switch at low asset values. However, to reduce switching frequency decision makers try to increase the distance between the switching points $S_L$ and $S_H$. 

23
Figure 5. Location of the critical thresholds for different values of the high risk technology’s volatility $\sigma_H$. Higher risk increases the chances of an insolvent bank to gain solvency again and the bank is kept alive by equityholders for a longer time ($B^*$ decreases). Higher risk also makes banks switch to high risk earlier at $S_H$. Higher risk will also make managers switch back to low risk ($S_L$) later, as the option value of the deposit insurance decreases with volatility. If $\sigma_H$ is above 0.22, equityholders will not switch back to low risk any more.

### A. Welfare analysis

In this section we focus on welfare aspects by exploring potential benefits from VaR regulation for the regulating agency as well as gains to financial institutions.

As the deposit insurance guarantees the face value $c/r$ to the bank’s depositors, the current value of the potential future liability ($DI$) of the deposit insurance corporation is given by the difference between the face value and the current market value of deposits. Figure 6 shows the regulator’s liability as a function of $V$. The liability under VaR regulation is always lower, showing that moving to Value-at-Risk regulation is beneficial to the regulator. This improvement is caused by the fact that bank managers switch back to low risk when the bank
Figure 6. The current value of the potential liability of the deposit insurance corporation for different levels of $V$ under Basle Accord and VaR regulation. For both choices of asset risk the regulator’s liability under VaR regulation is below the one under BA regulation. Under Value-at-risk regulation the deposit insurance claim forms a hysteresis, i.e., depending on history bank managers either choose high risk $\sigma_H$ or low risk $\sigma_L$.

is solvent. This reduces the risk in the banking sector and thus lowers the liability of the deposit insurance. However, troubled banks are still a threat for the regulator under both regulatory regimes. Independent of $V$ the regulator is better off when imposing a Value-at-Risk based regulation.

To analyze the impact of the regulatory regimes on the equityholder’s claims, Figure 7 shows the equity value for a solvent bank under both regulatory regimes. We see that equityholders of the solvent bank will gain from moving towards VaR regulation. This is remarkable considering the fact that the VaR regulated bank optimally switches between the two risk levels loosing the dead weight switching costs (which is 1% of the asset value any time the portfolio is restructured). The gain in equity value results directly from the reward in form of lower capital requirements for low risk banks. This outweighs the potential gain from increasing the
Figure 7. Equity value of the solvent bank ($V = 4000$) as function of the volatility $\sigma_H$ of the high risk technology. Value-at-Risk regulation gives solvent banks a higher equity value than regulation according to the Basle Accord.

value of the deposit insurance put-option. For reasonable values of the high risk technology – when offered two regulatory mechanism for choice – solvent banks will voluntarily vote for Value-at-Risk regulation. This reduces overall risk in the banking industry, benefits the regulator and is also increasing shareholder wealth. Thus, we find that there is potential for Pareto improvements when introducing true risk based bank regulation.

B. Prudent Regulation

While Value-at-Risk can reduce the risk in the banking industry and simultaneously increase shareholder value, some caution has to be applied when designing a regulatory mechanism. Under both regimes examined above gambling for resurrection by troubled banks cannot be
Figure 8. Minimum audit intensity $\lambda_{\text{min}}$ required to maintain the switching incentive under Value-at-Risk regulation for different volatilities $\sigma_H$ of the high risk technology. For higher risk levels, more intense auditing has to be provided. At very low levels of risk, banks managers are reluctant to switch to avoid switching costs and because the reduction in capital requirements is small.

prevented. Regular audits are therefore necessary to detect and subsequently close insolvent banks.

The pareto improvement found results mainly from solvent bank managers switching back to low risk because of the reward in form of lower capital requirements. When designing a regulatory mechanism $\langle \lambda, B(\sigma) \rangle$ the regulating authority cannot focus on the capital requirements alone but must also set a corresponding audit intensity to preserve the switching incentive. This minimum audit intensity must be related to the asset volatility of the high risk technology.

Figure 8 shows the minimum audit intensity $\lambda_{\text{min}}$ for VaR regulation that has to be implemented plotted against different volatilities of the high risk technology. As we can see,
more intensive auditing has to be provided as the high risk volatility increases, to maintain the beneficial effect of Value-at-Risk regulation.\textsuperscript{14} The importance of adequate auditing is also recognized by regulators, i.e., the Basle Committee on Banking Supervision (1999) suggests three pillars of a new capital adequacy framework: “minimum capital requirements, a supervisory review process, and effective use of market discipline”.

Two main policy implications can be derived from our analysis. First, when comparing Basle Accord and Value-at-Risk based capital requirements, we illustrate that the latter can be pareto superior if the regulator enforces capital requirements by auditing banks at more than the minimum audit intensity. Second, this minimum audit intensity for VaR regulation increases with the volatility of the risky technology, i.e., the regulator has to adjust the auditing process to the particular investment opportunities of the bank. One way of overcoming this drawback would be to make capital requirements more sensitive to the financial institutions risk\textsuperscript{15}. This would punish high risk banks more and enhance the risk reduction effect.

VI. Conclusion

We analyze the effect of alternative regulatory mechanisms on the risk taking incentives for banks as well as on the liability of the deposit insurance and the value of bank equity. We extended the results from previous studies by allowing bank mangers to switch between two different levels of risk. This explicit treatment of the risk shifting process allows the comparison of regulations, that are based on asset value – like the Basle Accord – and risk-contingent regulations – like Value-at-Risk based capital requirements. The main finding is, that the reward in form of lower capital requirements for low risk banks under risk based regulation gives management an incentive to reduce risk. This risk reduction decreases the liability of

\textsuperscript{14}If the difference between the volatility of the two technologies becomes too small, the switching costs outweigh the gain from lower capital requirements and managers will only switch when auditing is getting tougher.

\textsuperscript{15}A panic factor greater than one will accomplish this, however, the general upward shift of capital requirements might scare off equityholders.
the deposit insurance fund while the lower capital requirements increase the value of bank equity. Value-at-Risk regulation can therefore be pareto superior to Basle Accord rules. These observations are consistent with the recent proposals of bank regulators to let banks use their own models to determine capital requirements and with the increasing number of banks taking advantage of this “internal models approach”.

The analysis also shows, that excessive risk taking by financially distressed banks cannot be prevented under both regulatory regimes. To maintain the risk reduction incentive adequate auditing has to be provided by the regulator. The minimum audit intensity increases with the volatility of the high risk technology. To overcome this drawback, future capital standards should be designed such that the audit intensity is independent of the banks investment opportunities. One way to achieve this is to make capital requirements more sensitive to asset risk than under current Value-at-Risk regulation.
References


Basle Committee on Banking Supervision, 1996, Overview of the amendment to the Capital Accord to incorporate market risks, http://www.bis.org/publ/bcbs23.htm.


A. Valuing a Claim Contingent on \((V, \sigma)\)

A. The Market Value of Deposits

As long as the bank is alive there is a constant profit flow \(c\). In case of closure the value of the claim is \(\min\{V, c/r\}\). In terms of the general claim \(F\) we in Section II the market value of deposits determines the parameters \(\alpha, \beta\) and \(\gamma\) to

\[
\begin{align*}
\alpha &= c, \\
\beta &= 1_{[c/r, \infty)} c/r, \\
\gamma &= 1_{[0, c/r]} 1.
\end{align*}
\]

The market value of debt in an interval of stable regime can be written as

\[
D(V, \sigma; A_1, A_2) = \begin{cases}
\frac{c}{r+\lambda} + \lambda \left(1_{[c/r, \infty)} \frac{c}{r+\lambda} + 1_{[0, c/r]} \frac{1}{r+\lambda} V\right) + A_1 V^{x_1(\sigma)} + A_2 V^{x_2(\sigma)} & : V \leq B(\sigma) \\
\zeta + A_1 V^{y_1(\sigma)} + A_2 V^{y_2(\sigma)} & : V > B(\sigma)
\end{cases}
\]

The boundary conditions at the different bounds of stability are

1. If \((V_0, \sigma_c)\) is a switching threshold:

\[
\lim_{V \to V_i} D(V, \sigma; A_1, A_2) = D(V_0, \sigma_-; A_1', A_2').
\]

2. If \(V_f\) is a bankruptcy trigger:

\[
\lim_{V \to V_f} D(V, \sigma; A_1, A_2) = \min\{V, \frac{\zeta}{r}\}.
\]
3. If $V_i$ is the bound of the closure region, i.e., $V_i = B(\sigma)$:

\[
\lim_{V \to V_i^-} D(V, \sigma; A_1, A_2) = \lim_{V \to V_i^+} D(V, \sigma; A'_1, A'_2),
\]

\[
\lim_{V \to V_i^-} D_i(V, \sigma; A_1, A_2) = \lim_{V \to V_i^+} D_i(V, \sigma; A'_1, A'_2).
\]

4. If $V_i = c/r$:

\[
\lim_{V \to V_i^-} D(V, \sigma; A_1, A_2) = \lim_{V \to V_i^+} D(V, \sigma; A'_1, A'_2),
\]

\[
\lim_{V \to V_i^-} D_i(V, \sigma; A_1, A_2) = \lim_{V \to V_i^+} D_i(V, \sigma; A'_1, A'_2).
\]

5. If the current regime is unbounded from above:

\[
\lim_{V \to \infty} D(V, \sigma; A_1, A_2) = \begin{cases} 
\frac{c}{r+\lambda} + \lambda \left( \frac{c}{r+\lambda} \right) & : B(\sigma) = \infty \\
\xi & : B(\sigma) \neq \infty
\end{cases}
\]

If it is unbounded from below:

\[
\lim_{V \to 0} D(V, \sigma; A_1, A_2) = \begin{cases} 
\frac{c}{r+\lambda} & : B(\sigma) \neq 0 \\
\xi & : B(\sigma) = 0
\end{cases}
\]

B. The Value of Tax Benefits

The advantage of debt is the fact that coupon payments to the debtholders are tax deductible, i.e., as long as the bank is alive there is a profit flow to the banks equityholders of the magnitude $\tau c$. In the case of bankruptcy, this tax shield is irretrievably lost. Therefore, the parameters $\alpha$, $\beta$, and $\gamma$ which characterize this claim are

\[
\alpha = \tau c, \\
\beta = 0, \\
\gamma = 0.
\]
The market value of tax benefits in an interval of stable regime can be written as

\[ TB(V, \sigma; A_1, A_2) = \begin{cases} \frac{r_c}{r + \kappa} + A_1 V^{x_1(\sigma)} + A_2 V^{x_2(\sigma)} & : V \leq B(\sigma) \\ \frac{r_c}{r} + A_1 V^{y_1(\sigma)} + A_2 V^{y_2(\sigma)} & : V > B(\sigma) \end{cases} \]  \hspace{1cm} (30)

The boundary conditions at the different bounds of stability are

1. If \( (V_i, \sigma_c) \) is a switching threshold:

\[ \lim_{V \to V_i} TB(V, \sigma; A_1, A_2) = TB(V_i, \sigma_c; A_1', A_2'). \]  \hspace{1cm} (31)

2. If \( V_i \) is a bankruptcy trigger:

\[ \lim_{V \to V_i} TB(V, \sigma; A_1, A_2) = 0, \]  \hspace{1cm} (32)

3. If \( V_i \) is the bound of the closure region, i.e., \( V_i = B(\sigma) \):

\[ \lim_{V \to V_i^-} TB(V, \sigma; A_1, A_2) = \lim_{V \to V_i^+} TB(V, \sigma; A_1', A_2'), \]

\[ \lim_{V \to V_i^-} TB(V, \sigma; A_1, A_2) = \lim_{V \to V_i^+} TB(V, \sigma; A_1', A_2'). \]  \hspace{1cm} (33)

4. The value \( c/r \) does not change the functional form of the payoff one receives in case of closure, thus it is not a bound of stable regime.

5. If the current regime is unbounded from above:

\[ \lim_{V \to \infty} TB(V, \sigma; A_1, A_2) = \begin{cases} \frac{r_c}{r + \kappa} & : B(\sigma) = \infty \\ \frac{r_c}{r} & : B(\sigma) \neq \infty \end{cases} \]  \hspace{1cm} (34)

If it is unbounded from below:

\[ \lim_{V \to 0} TB(V, \sigma; A_1, A_2) = \begin{cases} \frac{r_c}{r + \kappa} & : B(\sigma) \neq 0 \\ \frac{r_c}{r} & : B(\sigma) = 0 \end{cases} \]  \hspace{1cm} (35)
C. The Current Value of Switching Costs

The claim $SC$ denotes the current value of future switching costs, i.e., in the case of a switch at a threshold $V_i$ the immediate expenditure of $kV_i$ is required. The remaining characteristics of this claim are

$$\begin{align*}
\alpha &= 0, \\
\beta &= 0, \\
\gamma &= 0.
\end{align*}$$

(36)

The market value of the switching-cost claim in an interval of stable regime can be written as

$$SC(V, \sigma; A_1, A_2) = \begin{cases} 
A_1 V_{\sigma_1} + A_2 V_{\sigma_2} & : V \leq B(\sigma) \\
A_1 V_{\sigma_1} + A_2 V_{\sigma_2} & : V > B(\sigma)
\end{cases}$$

(37)

The boundary conditions at the different bounds of stability are

1. If $(V_i, \sigma_c)$ is a switching threshold:

$$\lim_{V \to V_i^-} SC(V, \sigma_c; A_1, A_2) = SC(V, \sigma_{\leq c}; A_1', A_2') + kV_i.$$  

(38)

2. If $V_i$ is a bankruptcy trigger:

$$\lim_{V \to V_i^-} SC(V, \sigma; A_1, A_2) = 0.$$  

(39)

3. If $V_i$ is the bound of the closure region, i.e., $V_i = B(\sigma)$:

$$\lim_{V \to V_i^-} SC(V, \sigma; A_1, A_2) = \lim_{V \to V_i^+} SC(V, \sigma; A_1', A_2'),$$

$$\lim_{V \to V_i^-} SC_i(V, \sigma; A_1, A_2) = \lim_{V \to V_i^+} SC_i(V, \sigma; A_1', A_2').$$

(40)
4. The value \( c/r \) does not change the functional form of the payoff one receives in case of closure, thus it is not a bound of stable regime.

5. If the current regime is unbounded from above:

\[
\lim_{V \to \infty} SC(V, \sigma; A_1, A_2) = 0
\]

If it is unbounded from below:

\[
\lim_{V \to 0} TB(V, \sigma; A_1, A_2) = 0
\]