The Optimal Timing of the Transfer of Hidden Reserves in the German and Austrian Tax Systems – A Market-Based Approach with Special Attention to the Term Structure of Interest Rates and Interest Rate Risk

November 29, 2001

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The author would like to express his gratitude to Stefan Bogner, Christopher Casey and Michaela Schaffhauser-Linzatti for valuable remarks and discussions.
Abstract

In this paper, the optimal timing of hidden reserves transfers is derived with special attention to the term structure of interest rates and interest rate risk, and using well-known concepts from the field of finance. The paper presents one model under certainty and, as a generalization of this model, another model under interest rate risk. In both models, the criterion used for decision-making is the value of the right to transfer, which can be interpreted as the initial cost of a replicating/hedging strategy for tax payments incurred/saved. In the model under certainty, the net present value concept is used to derive the value of the right to transfer. The procedure used in the model under interest rate risk is a combination of flexible planning and the no-arbitrage approach common in derivatives pricing. It is shown that the right to transfer hidden reserves with flexible timing is equivalent to an American-style conversion option. In addition, the impact of term-structure volatility on the value of the right to transfer is analyzed. The technique presented in this paper can also be used to solve other timing problems resulting from trade-offs between early and late tax payments/tax benefits.

Keywords:

Earnings Management, Timing Option, Real Option, Term Structure of Interest Rates, Interest Rate Risk, Risk Management
1 Introduction

This paper deals with earnings management, which is an important aspect of accounting. In general, earnings are managed by taking advantage of the right to choose among various alternatives. Examples of this include the choice of depreciation method and timing/valuation leeway in connection with bad debt provisions or inventory valuation (e.g., see Francis/Hanna/Vincent (1996), Healy/Wahlen (1999) or Jackson/Pitman (2001)).

A large number of empirical studies have been carried out on the topic of earnings management to test various earnings management hypotheses involving different incentives. In addition, there are empirical studies on the optimal design of earnings management studies (see Guay/Kothari/Watts (1996), McNichols (2000), Bartov/Gul/Tsui (2000) or Thomas/Zhang (2000)). In comparison to the large quantity of empirical literature in this field, relatively few theoretical works have been written on earnings management. Examples of the latter include the work of Trueman/Titman (1988), Hand (1993), Demski/Frimor (1999) and Wagenhofer (1999).

This paper deals with a theoretical model for an earnings management problem encountered in German-speaking countries. In this context, therefore, it is useful to point out several differences between Anglo-American accounting and that of German-speaking countries. First of all, creditors' protection – as opposed to a purely informational function of the financial statements – is dominant in German-speaking countries, a fact that lends great importance to the principle of conservatism. Second, there is a link between commercial law and tax law in German-speaking countries.

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1 There are, for example, papers which investigate whether earnings management is performed in order to smooth earnings (Ronen/Sadan (1981)), in order to increase manager compensation (Healy (1985), Holthausen/Larcker/Sloan (1995), Balsam (1998), Soo (1999), Guidry/Leone/Rock (1999)), in order to reduce tax liabilities (Boynton/Dobbins/Plesko (1992), Hand (1993), Chen/Daley (1996), Chung (1998), Calegari (2000)), in order to fulfill regulatory requirements (Moyer (1990), Scholes/Wilson/Wolfson (1990), Collins/Shackelford/Wahlen (1995), Chen/Daley (1996), Kim/Kross (1998)), in order to avoid/delay technical defaults (Sweeney (1994)) or in order to fulfill management’s performance forecasts or the market expectations of analysts (Robb (1998), Kasznik (1999)). In addition, the connections between earnings management and takeovers (Easterwood (1997)), CEO changes (Pourciau (1993)), antitrust investigations (Cahan (1992)), import relief investigations (Jones (1991)), labor union negotiations (Liberty/Zimmerman (1986)), management buyouts (De Angelo (1988), Perry/Williams (1994)) and equity offers (Aharony/Lin/Loeb (1993), Teoh/Wong/Rao (1998)) have been examined.
which is not found in England or the USA, as taxable income is calculated on the basis of commercial earnings (with several corrections). This means that the area of earnings management that deals with minimizing tax liabilities is attributed great importance in German-speaking countries.

This specific area of earnings management is the focus of this paper. One essential research area within this form of earnings management is the analysis of timing decisions in light of their tax effects. This paper deals with one such timing decision for German and Austrian companies by analyzing the transfer of hidden reserves in particular. The background to this problem lies in the principle of conservatism and the resulting lower-of-cost-or-market principle, according to which an asset can not be valued above its acquisition costs. The lower-of-cost-or-market principle implies that an asset's book value can be below its market value. This circumstance gives rise to the tax-relevant "disclosure" of hidden reserves when the asset is sold. German and Austrian tax laws (§6b and §6c of the German Income Tax Act [EstG] and §12 of the Austrian Income Tax Act [EstG]) make it possible for companies to transfer such hidden reserves (disclosed when an asset is sold) to another, newly purchased asset as an expense within a certain period of time (subsequently called the "transfer period" in analogy to the Austrian EStG), and this strategy is used frequently in practice (see Hilke (2000), S. 122).3

The purpose of this paper is to determine the optimal point in time for transfer, with special attention to the term structure of interest rates and uncertainty regarding future interest rate evolution. As regards the optimal transfer strategy (without referring to the term structure of interest rates or interest rate risk), for example, Seicht (1990) writes that this is a considerable strategic opportunity in accounting policy.

However, literature on the transfer of hidden reserves is sparse. Many textbooks (e.g., see Endriss (1998)) implicitly assume a fixed time of transfer and suggest transfers to assets with the longest useful life in order to postpone tax payments as long as possible. Wittmann (1982) as well as Euler (1984) examine the benefits of allocating §6b

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2 For a detailed description of differences between American and German accounting, see Corbridge/Austin/Lemak (1993). Accounting in other German-speaking countries is relatively similar to German accounting.

3 Similar possibilities, albeit with restricted flexibility, can be found in R.35 of the German Income Tax Directives and Art. 64 of the Swiss Federal Act on the Direct Federal Tax. Both of these legal sources allow - under certain prerequisites - businesses to transfer hidden reserves disclosed by the sale of assets into a substitute investment or to allocate a provision or reserve for replacement investments.
reserves and the optimal liquidation strategy for §6b reserves, but both studies are based on the assumption that no transfer or reinvestment is initially planned.

Von Rosenberg/Müller (1990) analyze whether allocating a §6b reserve makes sense in cases where reinvestment is not planned / not certain. In a *ceteris paribus* analysis, these authors demonstrate that a transfer becomes more advantageous the later it is performed ("time effect"), the longer the useful life of the new asset is ("useful life effect"), and the higher the after-tax interest rate is ("interest rate effect"). In order to decide whether to allocate a § 6b reserve or not, the businessperson's personal risk attitudes have to be taken into account. The optimal time to liquidate the reserve (i.e., the optimal time of transfer) in consideration of the three influencing factors (time, useful life and interest rate) is not discussed.

This paper presents models for the German and Austrian tax systems for determining the optimal time of transfer. The approach taken here differs from previous works in the following ways:

First, the effects described by von Rosenberg/Müller (1990) are integrated into a model in order to determine the optimal transfer strategy. The optimal timing of the transfer is straightforward in cases where the investment project with the longest useful life starts at the latest possible transfer time (i.e., at the end of the transfer period). In such cases, this point in time prevails over all earlier dates. (Late transfer to an asset with a long useful life always postpones tax payments further into the future than an early transfer to an asset with a short useful life.) In all other cases, however, a trade-off exists between the time effect and the useful life effect, thus making optimization necessary. This paper presents a solution to this optimization problem in a coherent model.

The second significant expansion this paper adds to the existing literature is that well-known methods from the field of finance are applied in order to solve this timing problem. The replicating principle used (tax payments incurred/avoided are replicated by buying/selling bonds) has the advantage of enabling market-based decisions, thus making it unnecessary to quantify the businessperson's personal risk attitude.

Third, the model under certainty in this paper is the first to integrate the term structure of interest rates into the transfer decision. The model under certainty is then generalized

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4 See below for further details on §6b reserves.
to become a model under interest rate risk. This step is appropriate in light of the fact that interest rate volatilities have increased in recent years, which has also manifested itself in the growing number of interest rate derivatives and the increased attention to interest rate risk in the relevant literature (e.g., see Heath/Jarrow/Morton (1990), Bühler et al. (1999) or Frühwirth (2001)). In particular, the influence of term structure and interest rate risk on management decisions has been the subject of increased analysis (e.g., see Ingersoll/Ross (1992) or Höger (1995)).

The analysis in this paper comes to the following conclusions: In the model under certainty with attention to term structure, it is shown that the optimal transfer time generally depends on the initial term structure of interest rates and on the development of the investment program over time.

The model under interest rate risk demonstrates that the management's flexibility in timing the transfer is comparable to an American-style conversion option. Premature transfer of a hidden reserve can be optimal or sub-optimal, and a flexible transfer strategy (i.e., a stochastic transfer time) can represent the optimum. The approach used, which is based on options pricing theory, makes it possible to value the right to transfer in consideration of the flexibility with respect to transfer times. In this context, the influence of interest rate volatility on the optimal transfer strategy and on the value of the right to transfer is also examined.

This paper is structured as follows: Section 2 briefly describes the relevant legal regulations in Germany and Austria. Section 3 explains the model under certainty, a base model which is then supplemented by a generalization for the case of interest rate risk to be found in Section 4.

2 Legal Regulations in Germany and Austria

The possibility of transferring hidden reserves in Germany is governed by §6b and §6c of the German EStG. In §6b Paragraph (2), hidden reserves are defined as the amount by which the sale price of an asset exceeds its book value after deduction of the selling costs. Under §6b Paragraph (1), hidden reserves from the sale of land, crops along with the accompanying land (if the crops belong to operating assets used in agriculture/forestry), and buildings are transferable. Transfers can be made to land,
crops along with the accompanying land, or buildings. The requirements for such transfers are listed in Paragraph (4).

§6b Paragraph (3) allows companies to allocate an untaxed reserve in the year of hidden reserve disclosure for later transfer, unless they have transferred the hidden reserve in that year. In this way, the hidden reserve can be transferred to assets (described in Paragraph 1) acquired or produced in the ensuing four years. At the time of transfer, the untaxed reserve is to be liquidated.

§6b Paragraph (6) says that, if the hidden reserve is transferred to a new asset under Paragraph (1) or (3), the amount of the hidden reserve is to be subtracted from the full acquisition or production costs for tax purposes. The annual tax depreciation amount for the new asset is to be reduced accordingly.

In cases where the untaxed reserve still exists at the end of the fourth year, the legal consequences are described in two places: §6b Paragraph (3) dictates that a reserve which still exists at the end of the fourth year after its allocation is to be liquidated to increase earnings at that time. In such cases, §6b Paragraph (7) further requires that a premium be added to taxable earnings in the amount of 6% of the liquidated reserve amount for each full business year in which the reserve existed.

A comparable opportunity can be found in §12 of the Austrian EstG. In contrast to its German counterpart, this section does not provide a complete list of the assets to which hidden reserves can be transferred. Paragraphs 2 and 3 describe the circumstances under which such transfers are permissible.

For tax purposes, §12 Paragraph (5) stipulates a reduction of the acquisition/production costs of the new asset in a manner similar to German law: Tax depreciation is calculated on the basis of actual acquisition/production costs reduced by the amount of the transferred hidden reserve. Similar to the German law, §12 Paragraph (7) says that an untaxed reserve can be allocated in the year of disclosure unless the hidden reserve is transferred to a new asset in the same business year. Under §12 Paragraph (8), the period for the liquidation of this untaxed reserve is twelve months, or in certain cases 24 months, from the disposal of the asset.
According to §12 Paragraph (8), if the hidden reserve is not transferred to a new asset by the end of that period, the untaxed reserve has to be liquidated with an increase in profits. In contrast to Germany, Austria does not provide for a premium in cases where the reserve is liquidated without being transferred to a new asset.

3 Model under Certainty

The objective of this section is to develop a base model under certainty in order to determine the optimal transfer time with attention to the term structure of interest rates. This model will then be expanded to include interest rate risk in Section 4.

Section 3.1 describes the structure of the model. In Section 3.2, the (tax) payments relevant to decision-making are determined on the basis of the model, as is the value of the right to transfer the reserve at a given point in time. In Section 3.3, the net present value concept is used to identify the optimal time of transfer among all possible transfer times.

3.1 Structure of the Model

In this model, a capital market without transaction costs and with the possibility of short selling is assumed. On this market, there are infinitely divisible, credit-risk free, tax-exempt zero-coupon bonds with a par value of one currency unit for all maturities up to the model’s horizon $\tau$. The prices of zero-coupon bonds with various maturities reflect the interest rates of differing maturities and thus the term structure of interest rates.\[5\]

The model assumes a business whose profits are taxed at the tax rate $s$. The annual accounts are closed as of December 31 each year. Taxes are also to be paid on December 31.

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5 The assumptions in this paragraph are standard assumptions in after-tax bond valuation as well as after-tax capital budgeting, and they enable after-tax market-based valuation. These assumptions, especially that which says there are infinitely divisible, credit-risk free, tax-exempt zero-coupon bonds for all maturities up to the model’s horizon, are far less problematic than they may seem at first glance. Even if tax-exempt zero-coupon bonds were non-existent, they could be created synthetically using bonds subject to taxation with various maturities if an arbitrage-free, complete market is assumed (see Steiner/Uhlir (2000), p. 47ff). For further details on the estimation of the after-tax term structure of interest rates on an incomplete market, see McCulloch (1975).
It is assumed that the business in question sold fixed assets in the previous year at a price higher than their book value for tax purposes, whereby a hidden reserve in the amount of $SR$ was disclosed. Thus the business' taxable earnings rise by the amount of $SR$ at the end of that year (time: 0). If we assume that the business' taxable earnings for that year (before disclosure of the hidden reserve) are non-negative, the business' tax liability will increase by $SRs$ due to the disclosure.

It is also assumed that the legal prerequisites for transferring this hidden reserve are fulfilled. The business thus has the following alternatives at time 0:

1) The business can pay the taxes on the hidden reserve immediately. In this case, the previously mentioned increase in tax liability in the amount of $SRs$ at time 0 remains unchanged.

2) The hidden reserve $SR$ can be allocated to an untaxed "transfer reserve" at time 0 in order to transfer the hidden reserve immediately or within a period of $T$ years. The period $T$ is referred to as the "transfer period" below. This allocation to the transfer reserve reduces taxable earnings by $SR$ at time 0, resulting in tax savings in the amount of $SRs$. The decision to transfer the hidden reserve or at least to wait until the next year to do so is made at the end of each year. The effects of selecting this Alternative 2 in the ensuing years depend on whether or not the hidden reserve is actually transferred to a new asset within the transfer period, which is described under 2a and 2b.

2a) When the hidden reserve is transferred to a new asset, the transfer reserve is liquidated against the acquisition/production costs of the new asset, which in effect increases future tax payments (further details below). Selecting Alternative 2 therefore allows the business to postpone the increase in tax payments in the amount of $SRs$ from time 0 to the future, which brings about a corresponding interest gain.

2b) If the hidden reserve is not transferred within the transfer period, the transfer reserve is to be liquidated in its entirety to increase profits, with a percentage premium of $z$. This causes an increase in taxable earnings at the end of the transfer period in the amount of $SRs(1+z)$ and thus an increase in tax payments of $SRs$.

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6 This is an abbreviated description which is equivalent to actual entry procedures in its effects on tax liabilities. Because this special deduction for tax purposes is not actually depreciation under commercial law, it must not reduce the acquisition/production costs. For this reason, the hidden reserve transfer is actually entered into the books in the form of a "Special item with reserve portion" (in Germany) or a "Valuation reserve" (in Austria). Further details on the entry procedures for such transfers can be found in Endriss (1998) for Germany and in Wagenhofer (2000) for Austria.
The interest gain resulting from postponing the tax payment is thus counteracted by an increase in tax payments from $SRs(1+z)$. The interest gain resulting from postponing the tax payment is thus counteracted by an increase in tax payments from $SRs(1+z)$.

In order to shift tax payments into the future, the business decides for Alternative 2 at time $0$.\footnote{Under Austrian law, Alternative 2 is preferable to Alternative 1 in any case, because - even if the hidden reserve is not transferred during the transfer period - no premium is charged and therefore Alternative 2 will definitely result in an interest gain. Under German law, Alternative 1 may be preferable if the probability of transfer within the transfer period is very low and thus the probability of paying a premium for non-transfer in Alternative 2 is correspondingly high, which could more than compensate for the interest gain. However, the assumption that Alternative 2 is selected is necessary even in the German case, because it enables - along with the following model - a two-stage procedure for the decision between an immediate tax payment for the hidden reserve and the allocation of a transfer reserve: In the first stage, it is assumed that the hidden reserve is transferred (Alternative 2), and the model presented is used to calculate the value of the right to transfer based on the optimal transfer strategy. In the second stage, it is necessary to check whether the optimal strategy in Alternative 2 prevails over an immediate tax payment (Alternative 1).}

At time 0, the business’ investment program is already fixed up to the end of the transfer period. The program consists of investment projects with deterministic investment dates and deterministic useful lives for tax purposes. For the purposes of further analysis, only those projects to which the hidden reserve can be transferred are relevant. It is assumed that such a project exists at any given point in time.\footnote{The methodology described in this paper can likewise be used in the more general case of interest-contingent investment dates and/or interest-contingent useful lives. It is also possible to use a generalization in which no investment project is available at certain times or in certain states. At these times/in these states, other payments would be incurred in comparison to the times/states where investment projects are available, and the resulting conversion option described in Section 4.2 would become more complex. Integrating these generalizations, however, would increase the complexity of the model unnecessarily and not yield any essential additional results.}

In this paper, it is assumed that the hidden reserve can only be transferred to depreciable assets. This assumption is based on the highly restrictive legislation in Austria and Germany with regard to transferring hidden reserves to non-depreciable assets.\footnote{For example, the transfer of hidden reserves to land in Austria is only permissible for businesses which calculate their profits under §5 EstG and when the hidden reserves themselves arise from the sale of land. In Germany, similarly, the transfer of hidden reserves to land is only allowed when the profits arise from the sale of land. Integrating the right to transfer hidden reserves to non-depreciable assets into the model described would require knowledge of the time of sale for each of these assets, as the hidden reserves would be disclosed again at the time of sale.}

Due to the transfer of the hidden reserve, depreciation on the new asset is reduced, which implies that tax payments are shifted into the future. These tax payments are shifted farther into the future when the hidden reserve is transferred to assets with long useful lives than when the hidden reserve is transferred to assets with short useful lives.
Given a fixed time of transfer, a rational person would thus transfer the hidden reserve to the asset with the longest useful life. The maximum useful life over all assets to which the hidden reserve can be transferred at time \( t \) is deterministic but dependent on \( t \); this maximum useful life in years is represented by \( n(t) \). This time dependency is based on macroeconomic (business cycle, industry technology, etc.) and/or microeconomic (each business' investment program will fluctuate over time independently of macroeconomic factors) changes over time.

For each point in time, the following is assumed of the asset with the maximum useful life: The asset involves acquisition/production costs which exceed the amount of the hidden reserve that has been disclosed; the asset will be retained until the end of its useful life for tax purposes; tax depreciation is exclusively straight-line; and each year a full year's depreciation is claimed. Aside from the possible transfer of the hidden reserve to this asset, no other investment-related tax privileges are claimed, and no write-ups or write-downs are performed.

In summary, a transfer at time \( t \) creates, in effect, a reduction of the acquisition/production costs (for tax purposes) of the new asset by the amount of \( SR \). The transfer thus decreases tax depreciation by \( SR/n(t) \) at the end of each year from \( t \) to the end of the asset's useful life. It is assumed that in all tax years after time 0 the business' taxable earnings are sufficiently high that the decrease in tax depreciation increases tax payments by \( SR \times s/n(t) \).

### 3.2 Value of the Right to Transfer at Time \( t \)

The value of the transfer at time \( t \) can be calculated by determining the net present value of the tax payments which are incurred/avoided in the future due to the transfer at time \( t \). Table 1 gives an overview of the tax payments incurred/avoided in connection with

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10 These assumptions rule out, inter alia, the possibility that the hidden reserve might have to be transferred to several assets due to insufficient acquisition costs in the new asset, as well as the possibility that the transfer of hidden reserves to a new asset may bring about additional hidden reserves in the future (when the new asset is sold). In addition, interaction among investment-related tax privileges and/or write-ups/write-downs on the one hand and the transfer of the hidden reserve on the other hand is prevented. All of these assumptions could be relaxed within the framework of the model used, with the side-effect of increased complexity. Because no significant additional conclusions would arise from this increased complexity, this was not done.
the allocation of the transfer reserve at time 0 and with the transfer of the hidden reserve at time \( t \):

**Table 1: Tax payments incurred/avoided by allocating the transfer reserve/transferring the hidden reserve**

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Time points</th>
<th>0</th>
<th>( t, t+1, \ldots, t+n(t)-1 )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allocation of transfer reserve – Immediate tax savings</td>
<td>+ ( SR \cdot s )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Allocation of transfer reserve – Threat of tax-increasing liquidation at ( T ) if hidden reserve is not transferred by ( T )</td>
<td>- ( SR \cdot s \cdot (1 + z) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transfer – Avoidance of tax-increasing liquidation at ( T )</td>
<td>+ ( SR \cdot s \cdot (1 + z) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transfer – Reduction of depreciation amount during useful life of new asset</td>
<td>- ( SR \cdot s / n(t) )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Securing the transfer opportunity at time 0 generates the following payments: At time 0, a tax benefit in the amount of \( SR \cdot s \) results from the allocation of the transfer reserve. However, this implies a tax increase – including the premium – of \( SR \cdot s \cdot (1 + z) \) at the end of the transfer period in the case of non-transfer.\(^{11}\) Because the transfer opportunity is assumed to have already been secured (see Section 3.1), these payments are no longer relevant to decision-making.

The following payments are caused by transfer at time \( t \):

The tax increase of \( SR \cdot s \cdot (1 + z) \) for non-transfer at the end of the transfer period is avoided. The avoidance of this tax increase can be regarded as a cash inflow which compensates for the cash outflow in the same amount mentioned in the previous paragraph. The value of the avoided tax increase as of the time of transfer \( t \) can be

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\(^{11}\) In this paper, the term "tax increase" is used to denote any increase in tax payments.
calculated by multiplying it by the price of a zero-coupon bond with maturity $T$ at time $t$, which is denoted as $P_{t,T}$:

$$SR \cdot s \cdot (1 + z) \cdot P_{t,T}$$

On the other hand, the transfer at time $t$ increases tax payments in each year of the new asset's useful life by $SR \cdot s / n(t)$. The value of these increased tax payments in each year of the asset's useful life as of the time of transfer is $SR \cdot s \cdot \frac{RBF_{t,n(t)}}{n(t)}$, where $RBF_{t,n(t)} = \sum_{j=0}^{n(t)-1} P_{t,j}$. Borrowing from an analysis with a flat term structure of interest rates, $RBF_{t,n(t)}$ is referred to as a "present value factor for an after-tax annuity in advance" as of time $t$ for a maturity of $n(t)$ years.

The value of the transfer at $t$ is the value of the avoided tax increase at $T$ minus the value of the tax increases caused by the transfer in each year of the new asset's useful life (each as of the time of transfer $t$):

$$SR \cdot s \cdot (1 + z) \cdot P_{t,T} - \frac{SR \cdot s \cdot RBF_{t,n(t)}}{n(t)} = SR \cdot s \left[ (1 + z) \cdot P_{t,T} - \frac{RBF_{t,n(t)}}{n(t)} \right]$$

If, as of time $t$, the value of the tax increase avoided is lower than the value of the tax increases caused by the transfer, the value of the transfer at time $t$ will be negative. In such cases, deciding not to transfer the hidden reserve at $t$ and liquidating the transfer reserve to increase profits at the end of the transfer period is better, which means that tax payments are neither avoided nor incurred by transferring at time $t$. The value of the right to transfer at time $t$ is thus 0 instead of the (negative) value of the transfer at time $t$. The value of the right to transfer at time $t$ (as of time $t$) is generally:

$$V_{UM}(t) = \text{MAX} \left[ SR \cdot s \left[ (1 + z) \cdot P_{t,T} - \frac{RBF_{t,n(t)}}{n(t)} \right], 0 \right] = SR \cdot s \cdot \text{MAX} \left[ (1 + z) \cdot P_{t,T} - \frac{RBF_{t,n(t)}}{n(t)}, 0 \right].$$

### 3.3 Optimal Timing of the Transfer

In order to enable a comparison of various transfer times, a common point in time is necessary as a reference. The reference point in time used below is time 0.
valuation, as of time 0, of the individual values of the right to transfer at time \( t \), \( V_{\text{UM}}(t) \), is carried out using a market-based approach by multiplying values by the zero-coupon bond prices prevalent at time 0. The value of the right to transfer at time \( t \) as of the reference point 0 is therefore as follows:

\[
NPV_{\text{UM}}(t) = V_{\text{UM}}(t) \cdot P_{0,t} = SR \cdot s \cdot \max \left[ (1+z) \cdot P_{0,T} - \frac{TRBF_{t,n(t)}}{n(t)}, 0 \right]
\]

where \( TRBF_{t,n(t)} = P_{0,t} \cdot RBF_{t,n(t)} = \sum_{j=t}^{t+n(t)-1} P_{0,j} \), which can be regarded as the "present value factor for a deferred after-tax annuity in advance".

\( NPV_{\text{UM}}(t) \) can be interpreted as the additional amount which – given the transfer time \( t \) – remains for the enterprise/owner at time 0 if future tax payments (avoided/incurred due to the transfer at time \( t \)) are shifted to time 0 by buying/selling zero-coupon bonds at time 0.

The optimal time for transfer is then the time \( t \) which maximizes the value of the right to transfer at time \( t \) as of time 0, \( NPV_{\text{UM}}(t) \). The value of the right to transfer at any given point in time is thus:

\[
\max_{t \in \mathbb{S}} NPV_{\text{UM}}(t) = \max_{t \in \mathbb{S}} SR \cdot s \cdot \max \left[ (1+z) \cdot P_{0,T} - \frac{TRBF_{t,n(t)}}{n(t)}, 0 \right]
\]

Equation 1

This means that given the maximum useful life as a function of \( t \), \( n(t) \), one would tend to transfer later the more sharply \( TRBF_{t,n(t)} \) drops over time (i.e., when \( t \) is increased). Additional statements on the influence of level or slope of the term structure of interest rates are only possible with the use of numerical methods and would require knowledge of the maximum useful life as a function of \( t \), \( n(t) \).

**Example 1:**

The model horizon is four years starting from December 31, 2000 (\( t=0 \)). In the year 2000, hidden reserves in the amount of 100,000 were disclosed through the sale of fixed assets. Legal regulations permit the transfer of these hidden reserves. The transfer
period lasts two years, starting on December 31, 2000. No premium is charged for non-transfer within the transfer period. The business' tax rate is 50%.

The following transfer opportunities exist:

<table>
<thead>
<tr>
<th>Transfer time</th>
<th>Maximum useful life</th>
</tr>
</thead>
<tbody>
<tr>
<td>12/31/2000 (t=0)</td>
<td>( n(0) = 4 )</td>
</tr>
<tr>
<td>12/31/2001 (t=1)</td>
<td>( n(1) = 4 )</td>
</tr>
<tr>
<td>12/31/2002 (t=2)</td>
<td>( n(2) = 2 )</td>
</tr>
</tbody>
</table>

The term structure of interest rates as of December 31, 2000 is as follows:

<table>
<thead>
<tr>
<th>( j )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{0,j} )</td>
<td>1</td>
<td>0.97045</td>
<td>0.92682</td>
<td>0.87110</td>
<td>0.82531</td>
</tr>
</tbody>
</table>

The optimal transfer time can be calculated as follows: The value of the transfer on December 31, 2000 is

\[
50,000 \times \left[ 0.92682 - \frac{1 + 0.97045 + 0.92682 + 0.87110}{4} \right] = -764,
\]

which means that this cannot be an advantageous time to transfer. The value of the right to transfer on December 31, 2000 is thus 0. The value of the right to transfer on December 31, 2001 is

\[
50,000 \times \left[ 0.92682 - \frac{0.97045 + 0.92682 + 0.87110 + 0.82531}{4} \right] = 1420.
\]

The value of the right to transfer on December 31, 2002 is

\[
50,000 \times \left[ 0.92682 - \frac{0.92682 + 0.87110}{2} \right] = 1393.
\]

Therefore, in the decision between December 31, 2001 and December 31, 2002, the useful life effect prevails over the time effect. The optimal time for the transfer is December 31, 2001. By postponing the transfer from December 31, 2000 to December 31, 2001, the business gains 1420 currency units (CUs).

If the term structure of interest rates were subjected to an upward parallel shift (i.e., if zero-coupon bond prices were shifted downward), with \( P_{0,1} = 0.95983 \), \( P_{0,2} = 0.90665 \), \( P_{0,3} = 0.84282 \) and \( P_{0,4} = 0.78978 \), for example, then December 31, 2002 would be the optimal time to transfer the hidden reserve (1594 CUs for transfer on December 31, 2001 vs. 1596 CUs for transfer on December 31, 2002). If the term structure of interest rates were flatter than in the given example – e.g., \( P_{0,1} = 0.97045 \), \( P_{0,2} = 0.92867 \), \( P_{0,3} = 0.87634 \) and \( P_{0,4} = 0.83527 \) – then the optimal transfer time would likewise be shifted to December 31, 2002 (1299 CUs for transfer on December 31, 2001 vs. 1308 CUs for transfer on December 31, 2002). This gives an example of the influence of the level and slope of the term structure of interest rates on the optimal time of transfer. However,
generalizations of these statements for other maximum useful lives as a function of $t$ are not possible.

4 Model under Interest Rate Risk

The objective of this section is a generalization of the base model described in Section 3 for the case of interest rate risk. For this purpose, Section 4.1 gives a generalization of the model's structure, and Section 4.2 generalizes the valuation and optimization technique described in Section 3.

4.1 Structure of the Model

In addition to the securities described in the base model, a tax-exempt, credit risk-free money-market account exists.\(^{12}\)

Uncertainty arises from development in the prices of zero-coupon bonds with various maturities, that is, from the evolution of the after-tax term structure. The zero-coupon bond prices follow an arbitrary, discrete-time, complete, no-arbitrage term structure model (for the sake of simplicity, it is assumed below that exactly one interest rate change occurs per year\(^{13}\) and that the zero-coupon bonds are always traded on December 31). The term structure model can be assembled using any number of factors (sources of uncertainty). Recombining trees (path-independent models) as well as non-recombining trees (path-dependent models) are permissible. Using an arbitrary term structure model permits, for example, a model in which the term structure is subjected to parallel shifts (see Ho/Lee (1986)), or a model in which short-term interest rates fluctuate more drastically than long-term rates (see Heath/Jarrow/Morton (1990)).

Because a complete, no-arbitrage term structure model is assumed, there is a unique risk-adjusted probability measure under which the relative zero-coupon bond prices

---

\(^{12}\) This assumption is redundant because the money-market account can be replicated by means of revolving investments in single-period zero-coupon bonds. Nevertheless, it is used as a standard assumption in term structure models.

\(^{13}\) Because changes in interest rates during the year are realistic, it appears useful for the purposes of practical implementation to assume several interest rate changes per year, but only one time of transfer per year. This is possible within the framework of the model presented here, but because the purpose of this paper is to present the problem as well as an approach to be used as a basis for implementation, this is not done.
(zero-coupon bond prices in relation to the value of the money-market account) are martingales (see Harrison/Kreps (1979) and Harrison/Pliska (1981)).

Figure 1 gives a graphic overview of the evolution of the term structure over the first two years when a single-factor model is used.

**Figure 1: Evolution of the term structure of interest rates under a single-factor model**

Starting with node (0) at time 0, there are two nodes at time 1 – (1,U) and (1,D) – and four nodes at time 2 – (2,UU), (2,UD), (2,DU) and (2,DD) – where U stands for an upward movement and D for a downward movement in zero-coupon bond prices.
4.2 The Optimal Transfer Strategy

As in Section 3.3, the value of the right to transfer at time \( t \) as of time 0 is to be calculated on the basis of \( V_{U,M}(t) \), the value of the right to transfer at transfer time \( t \) as of time \( t \). In contrast to the model under certainty, it is now also possible to pursue flexible transfer strategies using stochastic transfer times. To provide an example in continuation of Example 1, the transfer strategy "If interest rates drop in the first year, we will transfer on December 31, 2002. If interest rates rise in the first year, we will transfer on December 31, 2001" represents one possible (stochastic) transfer time. Now that stochastic transfer times are also under consideration, valuation by simply multiplying by zero-coupon bond prices is no longer possible.

For this reason, a more general procedure which unites the advantages of flexible planning and the no-arbitrage approach to the valuation of contingent claims will be presented. The methodology is comparable to the one used by Ingersoll/Ross (1992) and Höger (1995) for the optimal timing of investment projects and has the advantage of enabling valuation on the basis of market prices, thus eliminating the need for assumptions regarding personal risk attitudes.

The procedure is based on the modeling of a no-arbitrage, complete market (see Section 4.1). On such a market, any payment can be replicated. At time 0, the payment must have the same value as the replicating portfolio. The value of the replicating portfolio (as of time 0) is the expected value under the risk-adjusted probability measure of the payment's present value, where the present value of the payment is calculated by dividing the payment by the value of a fictitious money-market account (e.g., see Pliska (1998)).

The value of the right to transfer – for a given transfer time \( t \) – as of time 0 is thus:

---

14 This also applies to payments incurred at a stochastic point in time, as long as this point in time fulfills the prerequisites of a stopping time (e.g., Pliska (1998), p. 134f; for prerequisites of a stopping time, see Lamberton/Lapeyre (1996), p. 17). These prerequisites are fulfilled for the transfer time because it is possible at any point in time \( t \) to determine whether the hidden reserve is transferred at \( t \).
\[ NPV_{UM}(t) = E_Q \left[ \frac{V_{UM}(t)}{B_t} \right] = E_Q \left[ \frac{SR \cdot s \cdot \text{MAX} \left( 1 + z \cdot P_{t,T} - \frac{RBF_{t,n(t)},0}{n(t)} \right)}{B_t} \right] \]

Equation 2

where \( B_t \) represents the value of the money-market account at time \( t \) \((B_0 = 1)\) and \( E_Q[.] \) depicts the expected value under the risk-adjusted probability measure.

The expression in Equation 2 can be interpreted as the amount which – given transfer time \( t \) – remains for the business(person) if a self-financing trading strategy in zero-coupon bonds is used to hedge the interest rate risk which results from the transfer at time \( t \) (and to which both the tax payments incurred during the new asset's useful life from \( t \) on and the tax payment avoided as of \( T \) are subject). In contrast to Section 3, a dynamic hedging strategy is generally necessary, i.e., bond transactions are necessary at all points in time.

The optimal transfer time is that which maximizes \( NPV_{UM}(t) \). The value (as of time 0) of the right to transfer at any time is therefore:

\[
\text{max}_{0 \leq t \leq T} NPV_{UM}(t) = SR \cdot s \cdot \text{MAX}_{0 \leq t \leq T} E_Q \left[ \frac{\text{MAX} \left( 1 + z \cdot P_{t,T} - \frac{RBF_{t,n(t)},0}{n(t)} \right)}{B_t} \right]
\]

Equation 3

This value corresponds to that of \( SR \cdot s \) American-style options with an intrinsic value at \( t \) of:

\[
\text{MAX} \left( 1 + z \cdot P_{t,T} - \frac{RBF_{t,n(t)},0}{n(t)} \right).
\]

From the definition of \( RBF_{t,n(t)} \) one can see that this is the intrinsic value of an American-style option to convert an equally weighted bond index into \((1+z)\) zero-
coupon bonds with maturity $T$. The bond index is calculated using a pool of $n(t)$ zero-coupon bonds with maturities from 0 to $n(t)$-1. The size of the pool is thus time-dependent and depends on the maximum useful life of all assets to which the hidden reserve can be transferred at $t$. The weighting assigned to each bond in the index is the reciprocal of this useful life.

The expression for the intrinsic value can be interpreted as follows: If $(1 + z) \cdot P_{t,T} > \frac{RBF_{t,n(t)}}{n(t)}$, then the option is in the money, and transferring at time $t$ is better than liquidating the transfer reserve (without a transfer) at the end of the transfer period $T$ (which does not necessarily imply that there is no transfer time that is better than $t$). If $(1 + z) \cdot P_{t,T} \leq \frac{RBF_{t,n(t)}}{n(t)}$, then the option is at (=) or out of (<) the money, and transferring at $t$ is equally advantageous (=) or worse (<) than liquidating the transfer reserve at $T$, meaning that a premature transfer at time $t$ is disadvantageous and the intrinsic value of the option is 0 (see Section 3.2).

The value of this option and the optimal exercise time depend on the initial term structure of interest rates, the maximum useful life as a function of $t$, $n(t)$, and on term structure dynamics. The optimal time to transfer the hidden reserve is identical to the optimal time to exercise the American-style option.

The calculation of the expected value in Equation 3, as well as the identification of the optimal exercise/transfer time, is possible using stochastic, dynamic optimization. One suitable procedure is backward recursion, in which one begins with the last possible point in time and then works back in succession to time 0 (e.g., see Cox/Ross/Rubinstein (1979), Ho/Lee (1986) or Jarrow (1996)).

Applying this procedure to the transfer of hidden reserves, the end of the transfer period is used as the point of departure. At this point in time, the value of the option, $V_T$, corresponds to the intrinsic value at $T$:

$$V_T = \text{MAX} \left[ (1 + z) \cdot P_{t,T} - \frac{RBF_{T,n(t)}}{n(T)}, 0 \right] = (1 + z) - \frac{RBF_{T,n(t)}}{n(T)} \geq 0$$
Because $z$ is non-negative and zero-coupon bond prices (under the plausible assumption of non-negative interest rates) are less than or equal to 1, $V_T$ is non-negative. It therefore makes sense to exercise the option in any case, at the latest at the end of the transfer period.

The values at arbitrary earlier points in time ($t < T$) can be calculated as follows:

The value upon exercise at $t$ is

$$V_{A,t} = (1 + z) \cdot P_{t,T} - \frac{RBF_{t,n(t)}}{n(t)}$$

The value at $t$, if exercise is postponed until after $t$, is

$$V_{W,t} = E_Q \left[ V_{t+1} \cdot \frac{B_t}{B_{t+1}} | F_t \right]$$

where $E_Q[.|F_t]$ is the expected value conditional on the information available at time $t$ (symbolized by $F_t$). (For more information on the concept of conditional expectation, see Duffie (1996), p. 272, for example.)

The value of the option at time $t$, denoted by $V_t$, is the maximum of the two values:

$$V_t = \text{MAX}[V_{W,t}, V_{A,t}]$$

The option is exercised at $t$ (i.e., the hidden reserve is transferred at $t$) if $V_{A,t} \geq V_{W,t}$. In all other cases the option is exercised later (i.e., the hidden reserve is transferred later).

---

15 This fact also allows analysis in the special case of a time-constant investment program (i.e., the maximum useful life is the same in all business years). Due to the martingale characteristics and the monotony of (discounted) zero-coupon bond prices, the end of the transfer period is the optimal time to transfer in this case. This is also intuitively plausible because the tax increases resulting from the reduced depreciation in each year of the new asset's useful life (the length of which is independent of the transfer time because the useful life is constant) can be postponed as far as possible into the future.
Example 2:

Complementing Example 1, this example assumes the following: The money-market account’s value as of December 31, 2000 is $B_0 = 1$. The time 0 bond prices used in Example 1 are taken as the point of departure:

<table>
<thead>
<tr>
<th>$j$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{0,j}$</td>
<td>1</td>
<td>0.97045</td>
<td>0.92682</td>
<td>0.87110</td>
<td>0.82531</td>
</tr>
</tbody>
</table>

On December 31, 2001 ($t=1$) two states are possible, one reflecting an upward movement ($U$) and the other reflecting a downward movement ($D$) in zero-coupon bond prices. In both states, the value of the money-market account is $B_1(U) = B_1(D) = 1/P_{0,1} = 1.03045$. Bond prices at these two nodes as of $t=1$ are shown in the table below:

<table>
<thead>
<tr>
<th>$j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{1,j}(D)$</td>
<td>1</td>
<td>0.92282</td>
<td>0.83744</td>
<td>0.76549</td>
</tr>
<tr>
<td>$P_{1,j}(U)$</td>
<td>1</td>
<td>0.97653</td>
<td>0.93775</td>
<td>0.90708</td>
</tr>
</tbody>
</table>

On December 31, 2002 ($t=2$), four states exist: $DD$ (downward movements in both periods), $DU$ (downward movement then upward movement), $UD$ (upward movement then downward movement) and $UU$ (upward movements in both periods). The value of the money-market account in states $DD$ and $DU$ is $B_2(DD) = B_2(DU) = B_1(D)/P_{1,2}(D) = 1.11664$. The value of the money-market account in the states $UD$ and $UU$ is $B_2(UD) = B_2(UU) = B_1(U)/P_{1,2}(U) = 1.05522$. Bond prices in these four states are shown in the table below:

<table>
<thead>
<tr>
<th>$j$</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{2,j}(DD)$</td>
<td>1</td>
<td>0.87686</td>
<td>0.77389</td>
</tr>
<tr>
<td>$P_{2,j}(DU)$</td>
<td>1</td>
<td>0.92789</td>
<td>0.86660</td>
</tr>
<tr>
<td>$P_{2,j}(UD)$</td>
<td>1</td>
<td>0.92789</td>
<td>0.86660</td>
</tr>
<tr>
<td>$P_{2,j}(UU)$</td>
<td>1</td>
<td>0.98190</td>
<td>0.97040</td>
</tr>
</tbody>
</table>

These term structure dynamics are identical to the development shown in Figure 1. They correspond to those in the Ho/Lee (1986) model with the parameters $q=0.6$ (risk-adjusted conditional probability for an upward movement of zero-coupon bond prices).
and $\delta = 0.945$ (a parameter which is inversely related to term structure volatility). The market is thus complete and arbitrage-free.

Now to the backward-recursive valuation of the option and to the identification of the optimal exercise/transfer strategy. Figure 2 shows the individual nodes in this model with the value of the option upon exercise ($V_{A,t}$), the value when exercise is postponed ($V_{W,t}$), and the value of the option ($V_t$).

Figure 2: Backward-recursive valuation of the option

Value calculations are explained in the order in which they are performed:
Step 1: Decision as of December 31, 2002 ($t=2$)

At the end of the transfer period, the value of the option is the value upon exercise. At node $(2,DD)$, this is:

$$V_2 = V_{A,2} = 1 - \frac{RBF_{2,2}}{2} = 1 - \frac{1 + 0.87686}{2} = 0.06157$$

The same procedure is to be applied to the remaining three nodes at $t=2$, resulting in the values shown in Figure 2.

Step 2: Decision as of December 31, 2001 ($t=1$)

Node $(1,D)$ is considered first. The value of the option if exercised after $t=1$ is:

$$V_{W,1} = E_Q \left[ V_2 \cdot \frac{B_1}{B_2} \mid F_1 \right] = 0.6 \cdot 0.03605 \cdot \frac{1.03045}{1.11664} + 0.4 \cdot 0.06157 \cdot \frac{1.03045}{1.11664} = 0.04269$$

The value upon exercise at $(1,D)$ is:

$$V_{A,1} = P_{1,2} - \frac{RBF_{1,2}}{4} = 0.92282 - \frac{1 + 0.92282 + 0.83744 + 0.76549}{4} = 0.04138$$

The value of the option at $(1,D)$ is thus 0.04269. Because $V_{W,1} > V_{A,1}$, the optimal strategy at this node is to postpone the exercise of the option.

An analogous procedure is applied to node $(1,U)$: The value of the option if exercised after $t=1$ is:

$$V_{W,1} = 0.6 \cdot 0.00905 \cdot \frac{1.03045}{1.05522} + 0.4 \cdot 0.03605 \cdot \frac{1.03045}{1.05522} = 0.01939$$

The value upon exercise at $(1,U)$ is:
The value of the option is thus 0.02119. Because $V_{A,1} > V_{W,1}$, the optimal strategy at node (1, $U$) is to exercise the option.

Step 3: Decision as of December 31, 2000 ($t=0$)

The value of the option if exercised at a later point in time is:

$$V_{W,0} = E_0 \left[ V_1 \cdot \frac{B_0}{B_1} \right] = 0.6 \cdot 0.02119 \cdot \frac{1}{1.03045} + 0.4 \cdot 0.04269 \cdot \frac{1}{1.03045} = 0.02891$$

The value of the option if exercised at $t=0$ is:

$$V_{A,0} = P_{0.2} - \frac{RBF_{0.4}}{4} = 0.92682 - \frac{1 + 0.97045 + 0.92682 + 0.87110}{4} = -0.01527$$

Thus the value of the option at $t=0$ is 0.02891.

Therefore, the optimal strategy at $t=0$ is to postpone the transfer of the hidden reserve. If bond prices rise by the time $t=1$, then the hidden reserve should be transferred. If bond prices fall by the time $t=1$, it is better to keep waiting until $t=2$ and then transfer the hidden reserve. According to Equation 3, the value of the right to transfer is 50,000 $\cdot$ 0.02891 = 1445 CUs.

This example shows that – even with a deterministic investment program – a flexible (i.e., interest-contingent) transfer strategy can be the optimum. In state (1, $U$), the useful life effect prevails over the time effect, which means that an earlier transfer to an asset with a longer useful life is preferable; in state (1, $D$), the opposite is true.

In comparison to Example 1, the value of the right to transfer the hidden reserve increases due to interest rate risk (1445 currency units as opposed to 1420 CUs).
When the volatility parameter $\delta$ is changed, the following results can be found:

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>Optimal transfer time</th>
<th>Node at which transfer is carried out</th>
<th>Value of right to transfer (in CUs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Example 1)</td>
<td>Deterministic ($t=1$)</td>
<td>$(1,\bar{U}), (1,\bar{D})$</td>
<td>1420</td>
</tr>
<tr>
<td>0.983</td>
<td>Deterministic ($t=1$)</td>
<td>$(1,\bar{U}), (1,\bar{D})$</td>
<td>1420</td>
</tr>
<tr>
<td>0.981</td>
<td>Stochastic ($t=1,t=2$)</td>
<td>$(1,\bar{U}), (2,DU), (2,DD)$</td>
<td>1421</td>
</tr>
<tr>
<td>0.945</td>
<td>Stochastic ($t=1,t=2$)</td>
<td>$(1,\bar{U}), (2,DU), (2,DD)$</td>
<td>1445</td>
</tr>
<tr>
<td>0.8</td>
<td>Stochastic ($t=1,t=2$)</td>
<td>$(1,\bar{U}), (2,DU), (2,DD)$</td>
<td>2281</td>
</tr>
</tbody>
</table>

The following is therefore true: A flexible transfer strategy (stochastic transfer time) is only optimal once a certain level of interest rate volatility has been exceeded. From this level upward, each increase in interest rate volatility leads to an increase in the value of the right to transfer.

Numerical analyses have also demonstrated such behavior for other term structures at time 0 and for other maximum useful lives as a function of $t, n(t)$: When volatility is close to 0 (i.e., $\delta$ is close to 1), a deterministic transfer strategy is always found to be optimal. When interest rate volatility rises, there are two possible scenarios, depending on the term structure at time 0 and the maximum useful life as a function of $t, n(t)$. For some combinations of initial term structure and $n(t)$, the optimal transfer strategy remains identical to the one under certainty for all volatility levels and the value of the right to transfer is then independent of interest rate volatility. For all other combinations, the following is true: Once a certain level of volatility is reached, a flexible transfer strategy becomes the optimum and each further increase in volatility beyond this level leads to an increase in the value of the right to transfer the hidden reserve. In addition to the three effects observed under certainty (time effect, useful life effect, interest rate effect), therefore, the model under interest rate risk also includes a "volatility effect" – which is definitely interesting from a taxation policy standpoint.

The connection between interest rate volatility and the value of the right to transfer ("The higher the interest rate volatility, the higher the value of the right to transfer") is reminiscent of the well-known fact from option pricing theory that the value of a European-style option rises with increasing volatility due to the convexity of its payoff profile. What is remarkable about this connection is that the conversion option
described here is American-style, not European-style, and that it is definitely exercised at some point in time and is thus *de facto* a forward contract, which causes its payoff profile to lose the convexity inherent to options.

**Summary**

In this paper, the optimal timing of hidden reserves transfers is derived with special attention to the term structure of interest rates and interest rate risk. The paper presents one model under certainty and, as a generalization of this model, another model under interest rate risk. In both models, the criterion used for decision-making is the value of the right to transfer, which can be interpreted as the initial cost of a replicating/hedging strategy for tax payments incurred/avoided.

In the model under certainty, the net present value concept is used to derive the value of the right to transfer. The procedure used in the model under interest rate risk is a combination of flexible planning and the no-arbitrage approach common in derivatives pricing. It is demonstrated that the right to transfer hidden reserves with flexible timing is equivalent to an American-style conversion option (between zero-coupon bonds maturing at the end of the transfer period and an equally weighted index composed of zero-coupon bonds with different maturities). In addition, the impact of term-structure volatility on the value of the right to transfer is analyzed.

The technique presented in this paper can also be used to solve other timing problems resulting from trade-offs between early and late tax payments/tax benefits.

**References**


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