# The Jarrow/Turnbull Default Risk Model Evidence from the German Market*

Manfred Frühwirth and Leopold Sögner

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**Abstract**

In this article we estimate default intensities within the continuous time Jarrow/Turnbull (1995) model from daily observations of German bank bond prices, based on the default-free term structure estimated from the Svensson (1994) model provided by the Deutsche Bundesbank. Cross-sectional and time-series estimations are performed. We show that pricing errors from the term structure estimation are passed on to the Jarrow/Turnbull model and that estimated default intensities strongly depend on the estimation period and on the bond used for estimation. This result strongly supports separate estimation over cross-sectional estimation. We cannot infer a significant influence of liquidity on the default intensities. We show that the assumptions of the Jarrow/Turnbull model are not sound with the underlying data. Furthermore, we investigate the optimum volume of data required to provide reasonable estimates of the default intensity. By this we show that daily calibration need not minimize the ex-ante pricing errors. Comparisons with other literature, where default-risky term structures are estimated separately from the default-free term structure, show that even using a very simple model the pricing errors can be decreased substantially.

Keywords: Credit Risk, Intensity-based models, Jarrow/Turnbull model, Term Structure of Interest Rates.

JEL codes: C52, G12, B13, E43.

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# 1 Introduction

One issue of growing importance both in academia and in industry is default risk. During the last decade, finance literature has put more attention to default risk by developing numerous credit risk models. The goal of this article is the estimation of the default intensity in the Jarrow/Turnbull (1995) model. We perform estimations of the default intensity based on default-free term structure data. The innovative aspect of this paper is to check for (i) the optimum volume of data required to derive reasonable estimates of the default probabilities, (ii) the impact of the underlying term structure estimation on the default intensity estimates, (iii) the plausibility of the constant default intensity assumption and (iv) the impact of liquidity on the default intensity. Our approach results in mean absolute pricing errors of 24.3 basis points (bp) for the cross-section and 5.0 bp to 19.3 bp for individual bonds. Compared to other literature, where the default risky term structure is estimated independently of the default-free term structure, pricing errors can be decreased substantially.

Let us start with the growing importance of default risk: The global credit quality has deteriorated and the volume of corporate bonds has risen dramatically over the last years. The number of defaults as well as the amount of defaulted debt have been increasing over the last years as has the number of rating agencies. Credit risk inherent in plain vanilla securities like sovereign bonds, stock options, interest rate or currency swaps or over-the-counter instruments, in general, has achieved greater attention (see e.g. Jarrow and Turnbull (1995), Duffie and Huang (1996), Duffie and Singleton (1997) or Coppes (1997)). Credit derivatives such as credit default swaps, total return swaps or credit spread options have created their market, growing dramatically over the last years.

This growing importance of credit risk has made the development of credit risk models an important issue in finance. Credit risk models can be categorized into *structural models* and *intensity-based models*. Within the first class, a firm value process is modeled while the latter models directly start with the default process. Beginning with the classical Merton (1974) model, structural models were presented by e.g. Black and Cox (1976), Brennan and Schwartz (1977), Geske (1977), Brennan and Schwartz (1980), Kim et al. (1993), Shimko et al. (1993), Nielsen et al. (1993), Leland (1994), Hull and White (1995) and Longstaff and Schwartz (1995). Despite the natural and elegant way of modeling default by the first time the firm value hits some barrier and although these models have proven very useful in investigating qualitative aspects of credit risk, this class of models has been criticized for several reasons.[1]

[1] Major points of criticism are the fact that in reality the firm value is non-traded and unobservable, the difficulty in modeling realistic boundary conditions, the predictability of default when the value of the underlying assets approaches the default boundary and problems in fully explaining the empirical behavior of credit spreads. For detailed discussions the reader is referred to Jones et al. (1984), Cooper and Martin (1996), Jarrow et al. (1997), Lando (1997), Madan and Unal (1998), Duffie (1998), Nandi (1998), Duffee (1999), Madan and Unal (2000) and Jarrow and Turnbull (2000).

Thus, an alternative model class has been created, the so-called *intensity-based models*. Such models have been presented e.g. by Litterman and Iben (1991), Lando (1994), Artzner and Delbaen (1995), Jarrow and Turnbull (1995), Das and Tufano (1996), Duffie and Singleton (1997), Lando (1997), Jarrow et al. (1997), Lando (1998), Madan and Unal (1998), Schönbucher (1998), Arvanitis et al. (1999), Jarrow and Yu (1999), Duffee (1999), Duffie and Singleton (1999), Duffie and Garleanu (2001) and Bakshi et al. (2001). Madan and Unal (2000) combine an intensity-based model with structural elements deriving the default intensity endogenously as the probability of losses exceeding equity. Duffie and Lando (2001) present a structural model consistent with a reduced-form representation including incomplete accounting information.

Intensity-based models restrict on observable variables, which facilitates empirical estimation. Moreover observed term structures of credit spreads can be fitted, a wide variety of dynamics for credit spreads can be handled in a tractable way and the models can be easily used for arbitrarily many classes of issuers.

This article covers the empirical implementation of the Jarrow and Turnbull (1995) model - which is a very simple model within the class of intensity based models - for German bank bonds. As Jarrow/Turnbull assume independence between the stochastic process driving the default-free term structure and the default process, term structure issues and default issues can be treated separately. The Jarrow/Turnbull model is built on an arbitrary Heath et al. (1992) term structure model, remaining analytically tractable. Another nice feature is the existence of explicit pricing formulas for risky bonds and for options on interest rate sensitive stocks which facilitates implementation and calibration.[2]

Let us relate our article to some current literature: Some empirical studies have analyzed structure, determinants and other features of credit spreads (see e.g. Sarig and Warga (1989), Litterman and Iben (1991), Longstaff and Schwartz (1995), Duffee (1998), Düllmann et al. (1998), Morris et al. (1999), Annaert et al. (2000) or Kiesel et al. (2001)). However, as argued by related literature, such as Nandi (1998), Duffee (1999), Skinner and Diaz (2000), Bühler et al. (2001) or Bakshi et al. (2001), only a limited amount of work has been done in empirically estimating and validating credit risk models. To fill this gap, some authors estimate intensity-based models (see e.g. Duffie and Singleton (1997), Duffee (1999), Düllmann and Windfuhr (2000), Bühler et al. (2001), Duffie et al. (2001), or Bakshi et al. (2001)).

In this paper we proceed as follows: We use term structures based on the Svensson (1994) model and provided by the Deutsche Bundesbank to describe the default-free term structure. Based on these term structures and on a recovery rate corresponding to Moody's studies, we derive estimates of the default intensity by means of non-linear least squares. Estimations are pursued for each of the bank bonds ("separate estimation") and for the cross-section. For both procedures, estimates are pursued at different days using prices at this particular day ("daily estimate") or prices observed within a certain time span

[2] For more detailed arguments in favor of the Jarrow and Turnbull (1995) model as opposed to other models see Jarrow and Turnbull (1995), Cooper and Martin (1996), Lando (1997), Nandi (1998) or Jarrow and Turnbull (2000).

("pooled estimate").

This analysis provides the following results: By comparing the default rate estimates under the empirical and the risk neutral measures respectively, we receive a Jarrow/Turnbull (1995) market price of default risk above one, which implies risk aversion with respect to default risk. When estimating the default intensity we observe considerable differences between different dates of estimation and different bonds, which strongly supports separate estimation. Using standard proxies for liquidity we find that liquidity has no significant impact on the default intensity estimates. However, the dependence of the default intensity estimates on the residuals from the default-free area is in most cases significant. From this, we conclude that the estimates of the default parameter depend on the default-free term structure estimation and that the structure of pricing errors from the default-free analysis are passed on to the estimates of the default intensity in the Jarrow/Turnbull (1995) model.

Default intensities are often extracted from daily data (see e.g. Jarrow et al. (1997), Schwaiger and Thym (1999) and Arvanitis et al. (1999) who calibrate default risk models with only few data points). One goal of our paper is to check whether these calibration techniques are justified in the Jarrow/Turnbull model. With term structure models, De Munnik and Schotman (1994) find that by weekly pooling of the corresponding bond prices reasonable estimates of the parameters can be derived. One aspect of our article is to investigate the optimum volume of data with both separate estimation and cross-sectional estimation of the default intensity. Our study shows that weekly pooled estimation improves the ex-ante fit when cross-sectional estimation is applied, with separate estimation the optimal pooling interval depends on the bond considered. The mean absolute pricing errors derived by pooling within the Jarrow/Turnbull framework are in the region of 5.0 to 24.3 basis points. Compared to the estimation of the default-risky term structure independent of the default-free term structure, performed with German data by Düllmann et al. (1998), the pricing errors decline substantially.

From separate estimation for individual bonds we observe that the estimated intensities differ substantially between different bonds. Therefore, either the information provided by the ratings is not sufficient, arbitrage opportunities exist in the market or the Jarrow/Turnbull model fails to reflect reality.

Related literature provides evidence, that there is correlation between credit risk and the default-free term structure of interest rates. Therefore, we perform a statistical analysis to check whether the Jarrow/Turnbull (1995) model assumption of a constant default intensity is sound with our data. Regressions show a statistically significant dependence on the slope of the default-free term structure in most cases. This implies that the Jarrow/Turnbull assumption of a constant default intensity cannot be supported.

This article is organized as follows. Section 2 gives a brief overview of the Jarrow/Turnbull model, Section 3 describes the data used. Section 4 details the estimation procedures applied and presents and discusses the results.

## 2 The Jarrow/Turnbull Model

Jarrow/Turnbull (1995) work in a frictionless, arbitrage-free and complete market setting. Firms are allocated to credit risk classes. Traded are default-free zero-coupon bonds as well as, in each risk class, default-risky zero-coupon bonds, for all maturities and each with a face value of 1. Jarrow/Turnbull consider a probability space with a filtration $(F_t)$. The corresponding empirical probability measure and the equivalent martingale measure (risk-neutral measure) will be abbreviated by $P$ and $Q$ in this paper. Default can occur at any time and comes as a surprise to all market participants. Default is modeled by a Poisson process stopping at the first jump. The corresponding stopping time is denoted by $\tau$ and will be called default time in the further analysis. The law of the *default process* is Poisson with intensity ("default rate", "hazard rate") $\lambda_1$, i.e. the law of $\tau$ is exponential with parameter $\lambda_1$. For each $t < \tau$, $\lambda_1 dt$ is the conditional probability of default in the time interval $[t, t + dt)$, having all information available up to $t$. Default is assumed to be independent of default-free interest rates. From bankruptcy on, each promised payment is reduced to a known fraction $\delta \in [0, 1]$ denoted as recovery rate, i.e. a default-risky zero-coupon bond with maturity $u$ pays 1 in $u$ iff $u < \tau$ and $\delta$ in $u$ iff $u \geq \tau$. This reduction is irreversible.

Following Heath et al. (1992), Jarrow/Turnbull (1995) model de/-fault-free zero-coupon bond prices by specifying the default-free forward rate process $f_0(t, u)$:

$$df_0(t,u) = \alpha_0(t,u)dt + \sigma(t,u)dW_1(t) \tag{1}$$

where $W_1(t)$ is a standard Brownian Motion under the empirical probability measure. The time $t$ prices of default-risky zero-coupon bonds with maturity $u$, $v\,(t,u)$, are modeled by means of default-risky forward rates $f_1(t,u) := -\frac{\partial Log[v(t,u)]}{\partial u}$ such that

$$\begin{aligned} df_1\,(t,u) &= (\alpha_1\,(t,u) - \theta\,(t,u)\,\lambda_1)\,dt + \sigma\,(t,u)\,dW_1(t) && iff\ t < \tau \\ &= (\alpha_1\,(t,u) - \theta\,(t,u)\,\lambda_1)\,dt + \sigma\,(t,u)\,dW_1(t) + \theta\,(t,u) && iff\ t = \tau \\ &= \alpha_1\,(t,u)\,dt + \sigma\,(t,u)\,dW_1(t) && iff\ t > \tau \end{aligned}$$

where $\theta(t,u)$ can be interpreted as positive adjustment to $f_1\,(t,u)$ which implies a price reduction occurring at default. Jarrow/Turnbull (1995) also provide the necessary condition for existence and uniqueness of the equivalent martingale measure $Q$: $\delta \exp(-\int_t^u \theta(t,\vartheta)d\vartheta) - 1 \neq 0$, $\forall\ t \leq \tau$, $u \leq \tau$ .

Furthermore, they assume that the default process is a Poisson process also under the equivalent martingale measure with intensity $\lambda := \mu\lambda_1$, where $\mu$ can be interpreted as a (constant) market price of default risk depending on the degree of risk aversion of market participants.[3] By solving for the price of a default-risky zero-coupon bond, $v(t,u)$, Jarrow/Turnbull derive

[3] We are aware that the "default intensity" $\lambda$, received by implicit estimation of the Jarrow/Turnbull (1995) model, is not a pure default intensity but it captures also all other factors that drive a wedge between corporate and government bond prices. These factors include e.g. any tax differences, liquidity differences and differences in systematic risk between corporate bonds and government bonds (see e.g. Grinblatt (1995), Duffie and Singleton (1997), Alessan-

$$
\begin{aligned}
v(t,u) &= p(t,u)\, E_Q\left[e(u)\,|\mathcal{F}_t\right] && (2)\\
&= p(t,u)[exp\left(-\lambda\left(u-t\right)\right)+\delta\left(1-exp\left(-\lambda\left(u-t\right)\right)\right)] && iff\ t<\tau\\
&= p(t,u)\delta && iff\ t\geq\tau
\end{aligned}
$$

where $p(t,u)$ is the time $t$ price of a default-free zero-coupon bond with maturity $u$. The variable $e(u)$ - being 1 for $u<\tau$ and $\delta$ for $u\geq\tau$ - can be interpreted as exchange rate since it expresses the relative price at $u$ of a default-risky to a default-free zero-coupon bond, both with maturities $u$. $E_Q\left[e(u)\,|\mathcal{F}_t\right]$ corresponds to the relative price at $t$ of a default-risky zero-coupon bond in terms of a default-free zero-coupon bond. Therefore, this term can be interpreted as discount factor for default risk. Equation 2 shows that the default-free term structure (i.e. zero-coupon bond prices), the default intensity and the recovery rate suffice to compute $v(t,u)$. Thus, the term structure *dynamics,* i.e. $\alpha_0(t,u)$ and $\sigma(t,u)$ from equation (1), apart from its influence on $p(t,u)$ don´t enter into $v(t,u)$ - a fact which is due to the independence assumption of Jarrow/Turnbull. Similar to the price of a default-free coupon bond the time $t$ price of a default-risky coupon bond with maturity $T$, $B(t,T)$, is a linear combination of its cash flows $C_B(u)$

$$
B(t,T)=\sum_{u=t}^{T}v(t,u)C_B(u)\ . \tag{3}
$$

Thus, to implement the Jarrow/Turnbull model, $p(t,u)$ as well as estimates of the default process parameters $(\lambda,\delta)$ are required. The following sections describe how this estimation can be performed and analyzes the problems appearing with the estimation.

# 3 Data

The data used are daily observations from January 1st, 1999 - which is the beginning of the currency union - to June 30th, 2000; $t=\frac{1}{365},...,\bar{t}$. Thus, within this analysis a step width of $\Delta=t_k-t_{k-1}=1/365$ is used. The data comprises 14 noncallable, nonputable, non-convertible, German Mark (DEM) denominated fixed coupon rate senior unsecured AA straight bonds[4] without

drini (1999), Huang and Huang (2000), Annaert et al. (2000), Liu et al. (2000), Prigent et al. (2001) and Elton et al. (2001)).

Although the bank bonds and government bonds we use are subject to identical tax treatment under the German tax system, it is to be analyzed in the future whether there are differences in systematic risk between German bank bonds and German government bonds. Furthermore, we consider liquidity differences as plausible. To get more insights into the role of liquidity, we investigate the influence of liquidity on the risk-adjusted "default intensity" in Section 4.2.1.

A more in-depth analysis of the factors behind the "default intensity" is not the purpose of this paper. (For a methodology to separate the individual components of the spread between corporate and government bonds see Liu et al. (2000) and Elton et al. (2001)). For the rest of this paper, we stick to the term "default intensity" keeping in mind that non-default factors might be subsumed among this term.

[4]We use the long-term rating from Standard and Poor´s.

sinking fund provision, issued by five big German banks[5] (Westdeutsche Landesbank GZ, Rheinland-Pfalz GZ, Bayerische Hypotheken- und Wechselbank, Deutsche Bank and Europäische Hypothekenbank der deutschen Banken). The five issuers remained AA over the whole observation period. For each issuer, except Europäische Hypothekenbank der deutschen Banken, we selected a short-term, a medium-term and a long-term bond with redemptions in 2001, 2003 and 2006.[6] The default-risky coupon bond prices observed are abbreviated by $B_i^{obs}(t,T_i)$, $i = 1,...,14$. As the data set comprises approximately 370 data points per bond, this results in about 5200 data points.

For the default-free area we use daily estimates of the parameters for the Svensson (1994) model. These parameters were estimated by the Deutsche Bundesbank. Furthermore, three government bonds with maturities 2001, 2003 and 2005 were used to get some insights into the mis-pricing of short-, medium- and long-term bonds using the Svensson parameters. For all bonds we use clean closing transaction prices from the Düsseldorf, the Frankfurt and the Munich stock exchange provided by Reuters Information Systems. For each bond and each day, we add accrued interest to the clean price to receive the dirty price.

# 4 Estimation and Results

For the estimation of the default intensity, the default-free zero-coupon bond prices $p(t,u)$ are required. Therefore, we use the following two-step procedure:

[5] An argument sometimes raised is that banks usually are bailed out and therefore cannot go bankrupt. However, bailing out is no default in the sense of Jarrow/Turnbull (1995) and other credit risk models, as no claims are reduced (see Jarrow et al. (1997): According to footnote 4 the term "bankruptcy" covers "any case of financial distress that results in the bondholders receiving less than the promised payment"). One recent and popular example of a bank default, corresponding to the usual definition, is the Barings case in 1995.

In addition, analysts forecast that intensifying competitive pressures in the banking sector will provoque an increase in the number of bank defaults. This argument is fostered as the European Shadow Financial Regulatory Committee requests national authorities to liquidate insolvent banks in order to reduce moral hazard problems inherent in bailing out and in order to achieve economic efficiency of the banking system (for these arguments: see Statements No. 4 and 5 of the European Shadow Financial Regulatory Committee from June and October 1999).

Furthermore, if banks could not default it should be argued why banking laws contain insolvency provisions for banks, why secured senior bonds or subordinated bonds are issued by banks and why there are price differences between government and bank bonds and within bank bonds of different seniority. It is hard to believe that the total of these price differences is attributable to liquidity differences.

Also, Kiesel et al. (2001) cannot support the argument, that bank bonds are less risky than industrial bonds with the same rating.

[6] Although the liquidity of bank bonds is higher than that of non-bank corporate bonds, it is smaller than the liquidity of government bonds. This problem sometimes leads researchers to estimate credit risk models from credit derivatives, e.g. credit default swaps, instead of corporate bonds (see Cossin and Hricko (2001)). As the credit derivative market in Germany is only in its infancy and therefore liquidity of credit derivatives is not satisfactory, we decided to estimate the Jarrow/Turnbull model using bank bonds. The same is done by Düllmann et al. (1998) and Houweling et al. (2001) who use German bond data of different rating classes for credit risk analysis.

We first compute default-free zero-coupon bond prices using the default-free term structure, estimated by the Deutsche Bundesbank according to the Svensson (1994) model. We assume that the estimated term structures of interest rates are the true term structures of interest rates. Based on this, we estimate the default intensity $\lambda$ by means of non-linear least squares using both cross-sectional estimation and separate estimation for individual bonds.

## 4.1 Default-free zero-coupon bond prices

For pricing default-risky interest rate derivatives and for risk management the estimation of the volatility function $\sigma(t, u)$ of the underlying Heath et al. (1992) term structure model (see equation 1) is necessary. However, for pricing corporate bonds with the Jarrow/Turnbull (1995) model and for evaluating the Jarrow/Turnbull model for bond pricing purposes, estimation of $\sigma(t, u)$ is not required. Since for bond pricing purposes only $p(t, u)$ is required (see equation 2) from the default-free area, the term structure time series suffices. This is one major advantage in the practical implementation of this credit risk model compared to more sophisticated models, like e.g. Jarrow/Turnbull (2000).

As our main goal is not a coverage of term structure issues but the estimation of the default intensity and the evaluation of the Jarrow/Turnbull model for bond pricing purpose, we waive the estimation of the parameters of the underlying default risk-free term structure model ($\alpha_0(t, u)$,$\sigma(t, u)$).

To get $p(t, u)$ we use Svensson (1994) parameters provided by the Deutsche Bundesbank. The parameter estimates are derived from government bonds and government notes with residual maturities of at least three months on a daily basis. These parameters can be downloaded from http://www.bundesbank.de. From these parameters and the Svensson function we can infer default-free spot rates and thus default-free zero-coupon bond prices $p(t, u)$ for all desired dates $t$ and maturities $u$. Using these zero-coupon bond prices to price the three government bonds, we derive a mean absolute pricing error of 14 basis points per day and bond.

## 4.2 Estimation of the default intensity

Following both industry (see e.g. Gupton et al. (1997)) and academic literature (see e.g. Lando (1994) or Altman and Kishore (1996)), the default intensity is determined by the issuer´s long-term senior unsecured credit rating whereas the recovery rate depends on the seniority of the bond. An implicit estimation of both recovery rate and default intensity results in highly unstable estimates. Therefore, we fix the recovery rate to $\delta = 0.5$, which is consistent with the recovery rate estimates for senior unsecured bonds in e.g. Moody´s (1992), Carty and Lieberman (1996), Altman and Kishore (1996), Standard & Poor´s (2000), Gupton et al. (2000) and Moody´s (2000).[7]

[7] We are aware that this procedure may cause distortions, since the $\delta = 0.5$ estimate is derived from the US market and there are differences between the German and the US bankruptcy legislation. Unfortunately, the limited number of default observations prohibits

The estimates of $\lambda$ at time t, $\widehat{\lambda}_t$, are derived from:

$$\widehat{\lambda}_t = \underset{\lambda}{argmin} \sum_{i=1}^{14} \sum_{j=t/\Delta-S+1}^{t/\Delta} \left[B_i^{obs}(j\Delta, T_i) - B_i(j\Delta, T_i)\right]^2 . \qquad (4)$$

Equation (4) computes the daily pricing errors between the Jarrow/Turnbull model price $B_i(j\Delta, T_i)$ and the observed market price $B_i^{obs}(j\Delta, T_i)$ and minimizes over the squared errors either by using all the senior unsecured bonds (*cross-sectional estimation*) or by using only one particular bond where $i \in \{1, ..., 14\}$ is fixed (*separate estimation*). We perform estimations using data from one day and using data from several days (*time-series estimation*) where $S$ is the number of days used in the estimation procedure. For $S = 1$ only the information of one day is used, while $S = 7$ - usually corresponding to 5 observations per bond - is referred to as "weekly pooled estimation"; $S = \bar{t}/\Delta$ corresponds to using data from all days within the observation period. If $S < \bar{t}/\Delta$, estimation (with pooling interval $S$) is done once a week.

Based on the estimated default intensities we perform linear regressions. In all these regressions we estimate the parameters of the model $y = \beta_0 + \beta_1 x_1 + ... + \beta_k x_k + \eta_t$ by means of ordinary least squares; $y$ is the response variable, $x_1, ..., x_k$ are the prediction variables, the intercept is $\beta_0$, $\beta_1$ to $\beta_k$ are the slopes, $\eta_t$ is the error term. $R^2$ denotes the coefficient of determination. Terms in parentheses refer to the t-statistics. A 5% confidence level is used within the statistical analysis.

Using all data available (cross-sectional estimation, pooling over the whole time span $S = \bar{t}/\Delta$) yields an estimate of the default rate of $\widetilde{\lambda}_{\bar{t}} = 0.0071$. Following the distinction of Jarrow et al. (2001) between a default risk premium and a market price of default risk the Jarrow/Turnbull (1995) model only deals with the market price of default risk, represented by the parameter $\mu$. From the one-year transition matrix from Standard & Poor´s (see e.g. Gupton et al. (1997)), we calculate the empirical default probabilities for various time horizons. Our calculation shows that in contrast to the Jarrow/Turnbull model the empirical default intensity is an increasing function of the time horizon. Using the estimate $\widetilde{\lambda}_{\bar{t}}$ together with these empirical default intensities the parameter $\mu$ has at least a value of 2 (where $\mu > 1$ implies risk aversion).

The following sections go into greater detail and check whether a global default intensity $\widetilde{\lambda}_{\bar{t}}$ is appropriate to reflect market prices of different bonds at different periods: Section 4.2.1 gains first insights into the differences between separate estimation and cross-sectional estimation. We derive estimates of $\lambda$

estimation of the recovery rates for German bonds based on historical default data. This forces us to rely on the assumption that regarding recovery rate the US experience is a good estimate for Germany. This also matches the Basle 2 provisions of a loss-given-default for bank loans of 50 % independent of the country. Furthermore, Moody´s (2001) state that there is no reason to assume that the 50 % rate doesn´t hold for Germany. Thus, when implementing their default risk model for Germany, they also use a 50 % rate. After contacting Riskmetrics we follow the advice to use a 50 % recovery rate for our study.

based on individual bonds. Based on this we check the content of information provided by the ratings and whether there is a rationale for cross-sectional estimation. As a byproduct, we investigate the influence of liquidity. Section 4.2.2 explores the impact of different pooling intervals on ex-ante pricing errors. This analysis is performed with both cross-sectional and separate estimation. By applying quadratic loss, the optimal pooling interval $\widehat{S}$ is derived. Based on $\widehat{S}$, Section 4.2.3 checks for the Jarrow/Turnbull model assumption of a constant default rate and determinants of the default rate.

### 4.2.1 Separate vs. cross-sectional estimation and the influence of liquidity

Figure 1 presents the separate estimates of $\lambda$ for each bond with $S = \bar{t}/\Delta$. Each group of three bonds relates to one issuer with the first bond being the short-term bond and the third bond being the long-term bond (for the order of the issuers see Section 3). Bond No. 13 is a short-term bond and bond No. 14 a long-term bond, both issued by Europäische Hypothekenbank der deutschen Banken.

[Figure 1 about here]

From Figure 1 it can be seen that the separate estimates for various bonds differ substantially within the same rating class or even for the same issuer. The smallest $\widehat{\lambda_{\bar{t}}}$ is equal to 0.0043 while the largest estimate equals 0.0091. From this point of view, either the information provided by the rating is not sufficiently detailed,[8] there exist mis-pricings in the market or the Jarrow/Turnbull model is not flexible enough to reflect market prices. Therefore, if we stay within the Jarrow/Turnbull economy, the user has the opportunity to believe either in the ratings or in an arbitrage-free world. In the former case, the credit risk manager should apply cross-sectional analysis in order to level out mis-pricings of individual bonds. In the latter case, only separate estimation for individual bonds should be performed as the rating does not serve us with sufficient information concerning default risk.

This section investigates whether cross-sectional or separate estimation should be performed. To increase the number of observations we split up the observation period into three half years. In order to detect determinants which might cause differences in the default rates – as documented in related literature - we perform a linear regression of $\hat{\lambda}_t$ on several variables: We use (i) bond specific variables, namely residual maturity (calculated at the beginning of the relevant half year), coupon and a purely bond specific dummy, $DUMMY_B$, (ii) an issuer specific dummy, (iii) two liquidity proxies and (iv) an estimation period

[8] There is some empirical evidence for rating being an insufficient criterion. E.g. Longstaff and Schwartz (1995) provide evidence that industry in addition to rating has an impact on the credit spread. Düllmann et al. (1998) come to the result that, in addition to rating, branch of industry, trading volume, coupon, and a "sticky rating" proxy are relevant for pricing credit risky bonds. Bakshi et al. (2001) find that firm specific factors and maturity are additional determinants.

specific dummy variable, $DUMMY_{HY}$, which indicates the corresponding half year. This dummy variable takes care for any time effects in the default intensity. We include maturity because Figure 1 suggests a positive relation between maturity and default rate which would be also consistent with e.g. Wei and Guo (1997). We include the coupon payment, since Düllmann et al. (1998) show a significant influence of the coupon on pricing errors, which should be reflected in our estimates. $DUMMY_B$ checks for additional bond specific differences. The issuer specific dummy should detect any issuer specific characteristics (e.g. smaller or larger default risk perceived by the market for some issuers) like in Bakshi et al. (2001).

Two arguments are raised for including liquidity proxies in our regression. First, as pointed out in footnote 3, the default intensity may also include a liquidity premium which should depend on the bond´s liquidity. Second, Ericsson and Renault (2001) find a positive correlation between liquidity risk and credit risk. Both facts should be reflected in higher default rates for less liquid bonds. We use issue size (in our sample between 20 Mil. DM and 400 Mil. DM) and age as proxies – for the following reasons: The amount outstanding and the age of a bond are two standard proxies for liquidity (see e.g. Warga (1992) and Ericsson and Renault (2001)). Since for our liquidity analysis we only use bonds where amount outstanding equals issue size, we can use issue size and age.[9] Larger issues are considered to be more liquid than smaller issues. Newly issued bonds are considered to be more liquid than older bonds. Thus, we would expect a negative coefficient for the size variable and a positive coefficient for the age variable in our regression.

In a first step we perform model selection recurring to the Schwarz Information Criterion. By this, coupon, maturity and the issuer specific dummy are excluded. In contrast to Bakshi et al. (2001), the issuer specific dummy is irrelevant. Therefore, a firm specific analysis is still to coarse to explain default intensities. The results of our regression are presented in Table 1. Surprisingly, neither liquidity proxy has a significant influence on the default intensity estimates. It can be seen that there is a positive significant dependence on the two remaining dummy variables. The significance of $DUMMY_B$ provides a strong argument for separate estimation. Nevertheless, for completeness and to get more insights into the ex-ante pricing quality of cross-sectional and separate estimates, we continue to cover cross-sectional estimation in the next sections. The significance of $DUMMY_{HY}$ suggests default intensities changing with time. We incorporate this time-dependence into the estimation methodology in the next section.

[9] We excluded the medium-term Deutsche Bank bond for the liquidity analysis, since for this bond the amount outstanding has been increased after issue. Although amount outstanding and/or age are frequently used as proxies (for a detailed survey see Annaert et al. (2000)) use of size as a proxy is questioned. Crabbe and Turner (1995) find no significant relation between size and liquidity, whereas e.g. Kempf and Uhrig-Homburg (2000) find that size is a reasonable proxy for liquidity. We decided to use both proxies since bid-ask spread or trading volume were not available and size and age are two proxies well established in Finance literature.

[Table 1 about here]

#### 4.2.2 Estimation based on limited information - The optimal pooling interval

Since section 4.2.1 suggests a default-intensity changing with time and the Jarrow/Turnbull model is not flexible enough to incorporate this changing default intensity, we provide an estimation procedure to handle this problem, in this section. The estimates of $\lambda$ are now updated once a week. The volume of data used to derive these estimates depends on the pooling interval $S$. I.e. the estimation procedure is based on pooling of (time series) data (see DeMunnik and Schotman (1994) for a similar procedure for term structure models). The following analysis investigates whether the cross-sectional and separate estimates differ significantly with the time span used for estimation and checks for the optimum of time series data to be used, i.e. we analyze the optimal $S$, to provide practitioners and researchers with a reasonable estimate of the default rate. By this, we also investigate whether calibration based on one day ($S = 1$), often performed in literature, is justified.

With the cross-sectional analysis we use all bonds. To investigate $S$, if a separate treatment of each bond is preferred, we provide three examples: the Westdeutsche Landesbank bonds with maturities $T_1 = 2001$, $T_2 = 2003$ and $T_3 = 2006$. In all cases considered we choose $S \in \{1, 7, 14, 21, 28, 35\}$ corresponding to daily, weekly pooled, biweekly, ... pooled estimates.

By defining a loss function (utility function), we are able to infer $S$ in a decision theoretic consistent way. In this analysis we use quadratic loss, resulting in the out-of-sample (i.e. ex-ante) mean squared pricing error per day and bond (MSPE). MSPE is derived from the pricing errors within the next week based on the estimate $\widehat{\lambda}_t$ from last Wednesday. For comparisons, we also investigate the MSPE for $S = \bar{t}/\Delta$ which is an in-sample (i.e. ex-post) error. Table 2 presents the results of this analysis, where $SD(\lambda)$ denotes the standard deviation of the estimates $\widehat{\lambda}_t$.

[Table 2 about here]

We observe that usually the standard deviation decreases when $S$ rises. For the cross-section as well as the short-term and medium-term West LB bond the MSPE shows a reduction if $S$ is raised at least from 1 to 7, i.e. if any pooling takes place. Although the MSPE with $S = \bar{t}/\Delta$ is an in sample error, it is always far larger than MSPE with optimal pooling. Thus, using limited information is superior (under quadratic loss) to using the whole time series. As regards the optimal pooling interval, $\widehat{S}$, we recommend $\widehat{S} = 7$ for the cross-sectional approach. For separate estimation $\widehat{S}$ depends on the specific bond ($\widehat{S} = 1$, 14 or 28). This shows that the optimal pooling interval has to be evaluated on a case-by-case basis.

Using the optimal pooling intervals we get an (ex-ante) mean absolute deviation between model price and market price of 24.3 basis points (bp) for the cross-section, 5.0 bp for the short-term West-LB bond, 11.2 bp for the medium-term West-LB bond and 19.3 bp for the long-term West-LB bond. Thus, using the optimal pooling intervals gives a reasonable fit. A comparison with Düllmann et al. (1998) suggests, even though we use a very simple credit risk model, a substantial improvement by model-based default-risky term structure estimation over an estimation of the default-risky term structures impendent of the default-free term structure. Düllmann et al. (1998) perform for the German market a separate term structure estimation for default-free and default-risky term structures based on Nelson and Siegel (1987) and get a mean absolute deviation for AA bonds of 50 bp. The fact that the cross-section fit is inferior to the fit from separate estimation, once more indicates that separate estimation is superior to cross-sectional estimation.

### 4.2.3 Determinants of the default rate

In a final step, we analyze the Jarrow/Turnbull assumption of a constant default rate for both cross-sectional and separate estimation. According to the previous section $\hat{\lambda}_t$ is provided by the estimates with the pooling interval resulting in the smallest MSPE. Therefore, following our results from Section 4.2.2, we select $\widehat{S} = 7$ for cross-sectional estimation, $\widehat{S} = 28$ for the short-term West LB bond, $\widehat{S} = 14$ for the medium-term West LB bond and $\widehat{S} = 1$ for the long-term West LB bond.

[Figure 2 about here]

Figure 2 shows the cross-sectional estimates as a function of time. Even from a visual inspection it is easy to see that these estimates are very volatile. To provide a test on the Jarrow/Turnbull assumption of a constant default rate, which is particularly independent of the term structure of interest rates, we run regressions where $\widehat{\lambda}_t$ is the response variable, while time, the one-year spot rate, the term spread (difference between the ten-year spot rate and the one-year spot rate) and the pricing error from the default-free analysis are the prediction variables.

We use time as prediction variable since $DUMMY_{HY}$ is significant in section 4.2.1 and the time variable checks for a linear drift in the default intensity. The one-year rate and the term spread are selected to check for a dependence on the default-free interest rates. The importance of this aspect is pointed out e.g. in Duffie and Singleton (1997). As indicated by several articles (see e.g. Litterman and Scheinkman (1991) or Duffee (1998)), most of the variation in the term structure can be captured by its level and its slope. Our proxy for the level is the one-year rate, calculated from the Svensson parameters. Basically, the choice of the maturity is arbitrary and we could have equally selected a long-term maturity as a level proxy (see Duffee (1998)). We did not use a spot rate for a maturity shorter than one year because term structure estimates usually are not reliable for too short maturities. Our proxy for the term spread is the difference between the ten-year spot rate and the one-year spot rate, both computed from the Svensson parameters. We did not select a spot rate with a maturity of more than ten years as both number and liquidity of government bonds is very low in the "$>$ 10 years" segment. The 10-year/1-year spread is also used as a proxy for the interest rate differential in the German debt market by the Deutsche Bundesbank.

Finally, we use pricing error time series from the default-free analysis (Svensson model price minus observed market price of the government bonds) to check for the influence of mispricings in the default-free area on the estimated default intensity. With the default intensity estimates from separate estimation, we synchronize the default-free bond with the bank bond used for estimation of the default intensity, i.e. for estimation from the short/medium/long-term bank bond we use the pricing error from the short/medium/long-term default-free bond. For the cross-sectional estimation we use the pricing error for the medium-term default-free bond as the cross-section is composed of short, medium and long-term bonds. Since for estimation date $t$ all pricing errors *within the pooling interval* up to $t$ have an influence on default-risky zero-coupon-bond model prices within the pooling interval and therefore on $\hat{\lambda}_t$, we use in all cases average pricing errors, denoted as $\varepsilon_t$, over the pooling interval. Table 3 presents the results of our regressions:

[Table 3 about here]

Except for the short-term West LB bond the default intensity significantly depends on the term spread (negative sign) and on the pricing error from the default-free analysis (positive sign).

The positive dependence on the pricing error is plausible because a positive pricing error implies that model prices (before default-risk adjustment) are too high which has to be compensated by a higher default intensity. This dependence shows that even if/just because the Jarrow/Turnbull setup allows for a separate treatment of term structure and default issues, a cautious treatment of the term structure is still necessary since a mis-specification of the default-free term structure influences the estimated default intensity.

The influence of the level of risk-free rates is indefinite (regarding the sign) and insignificant in our study (except for the short-term West LB). Literature on this determinant is mixed, too. E.g. Longstaff and Schwartz (1995), Wei and Guo (1997), Duffee (1998), Düllmann et al. (1998), Alessandrini (1999), Annaert et al. (2000) and Bakshi et al. (2001) come to the result that the credit spread is inversely related to the interest rate level. According to Duffee (1999) the influence of the risk-free rate level is not significant. Fridson and Jonsson (1995) and Jaffee (1975) find that the influence of the default-free term structure on credit spreads is negligable. Bühler et al. (2001) find an insignificant dependence on the default-free interest rate level (for most of the maturities investigated). Morris et al. (1999) show a negative short-run relationship and a positive long-run relationship. Arak and Corcoran (1996) find that parameters and significance depend on credit quality. Finally, Madan and Unal (2000) argue that the dependence of the default intensity on the default-free term structure level depends on the balance sheet structure, especially the duration gap, of the company observed.

The significantly negative influence of the term spread in our cross-section is in part consistent with Duffee (1998), Alessandrini (1999) or Annaert et al. (2000). In these papers a significant dependence of AA spreads on the term spread can be found - at least for some maturities. The signs of $\widehat{\beta}_3$ for the West-LB bonds are consistent with Düllmann et al. (1998) who find that a decrease in the slope of the risk-free term structure leads to a decrease in short-term credit spreads and an increase in medium- and long-term credit spreads; however $\widehat{\beta}_3$ in our study is insignificant at a 5% level for the short-term bond.

The signs of $\widehat{\beta}_3$ can be explained on the basis of empirical investigations of the term spread over the business cycle finding out that term spreads behave countercyclical (see e.g. Stambaugh (1988), Fama (1990), Estrella and Hardouvelis (1991), Harvey (1991), Harvey (1993), Mishkin and Estrella (1996) or Roma and Torous (1997)) combined with the hypothesis that an economic expansion reduces credit spreads (for the plausibility of this hypothesis see e.g. Arak and Corcoran (1996), Alessandrini (1999) or Annaert et al. (2000)).

Thus, if the term spread is high, for a short-term bond the remaining maturity is characterized by a trough (implying high empirical default probabilities), whereas for a medium-term or long-term bond an expansion (implying low empirical default probabilities) is expected to follow during maturity. Since the empirical default probability translates into the risk-adjusted default intensity, it is plausible that the term spread via the business cycle has a positive influence on default-intensities for short-term bonds, which turns into a negative influence on default-intensities for longer-term bonds.[10]

Thus, our study suggests that market participants in the German bond market use the term spread to forecast economic activity and the (empirical) default probability over a bond's life. This enters into the risk-adjusted default intensity reflected in market prices of corporate bonds. E.g. a high term spread parallels a trough and forecasts economic expansion and thus gives low (high) empirical default probabilities in the distant (near) future translating into low (high) risk-adjusted default intensities for long-term (short-term) bonds.

Summing up, we conclude from our analysis that the model assumption of a constant default rate is not supported by the data. Our study rather implies that research should turn to more flexible model specifications where the default rate is a function of some relevant parameters and the estimation of these models (see e.g. Arvanitits et al. (1999), Duffee (1999), Jarrow and Turnbull (2000) or Madan and Unal (2000) for more flexible models). Especially, the influence of the term spread on the default intensity should be explicitly modeled.

[10] Alternatively, economic activity could be proxied by a stock market index (see e.g. Jarrow and Turnbull (2000)). Including the DAX in our regressions results in a lower Schwarz Information Criterion. Therefore, only the term spread will be used in our analysis.

# 5 Summary and Conclusions

This paper presents various estimation methodologies for the model of Jarrow/Turnbull (1995), performing separate and cross-sectional estimation both using data from one day and using pooled estimates. Estimations are performed based on the Svensson (1994) term structure procedure for the default-free term structure.

Estimated default intensities depend on the default-free term structure estimation. Pricing errors from the term structure estimation are passed on to the Jarrow/Turnbull model. We show that the estimated default intensities depend on the date of estimation and the bond. The dependence on the bond implies that separate estimation from individual bonds must be supported. We show that liquidity has no significant influence. Another goal of our analysis is to determine the optimal volume of data required to estimate default intensities. If cross-sectional estimation is preferred, weekly pooled data should be used. For separate estimation the optimal pooling interval depends on the bond. Our approach results in mean absolute pricing errors of 24.3 bp for the cross-section and 5.0 bp to 19.3 bp for individual bonds. Comparisons with other literature, where default-risky term structures are estimated separately from the default-free term structure, show that even using a very simple credit risk model the pricing errors can be decreased substantially.

Another important finding of this paper is that the Jarrow/Turnbull hypothesis of a constant default rate does not fit the data, since regressions of the default intensity estimates show in most cases a statistically significant dependence on the slope of the term structure of default-free interest rates. Since this hypothesis cannot be supported, pricing errors due to mis-specification can arise from the application of the Jarrow/Turnbull model. Therefore, further research should engage in models where the default rate is a function of some relevant parameters and in the estimation of these models.

## 6 References

Alessandrini, F., 1999, Credit risk, interest rate risk, and the business cycle, Journal of Fixed Income 9 (2), 42-53.

Altman, E.I. and V.M. Kishore, 1996, Almost everything you wanted to know about recoveries on defaulted bonds, Financial Analysts Journal (Nov/Dec.), 57-63.

Annaert J, M.J.K. DeCeuster, F. DeJonghe, 2000, Modelling european credit spreads, Deloitte & Touche Research Report.

Arak M. and P.J. Corcoran, 1996, Yields on privately placed debt: Examining the behavior, Journal of Portfolio Management (Spring), 88-96.

Artzner, P. and F. Delbaen, 1995, Default risk insurance and incomplete markets, Mathematical Finance 5, 187-195.

Arvanitis, A., J.K. Gregory and J.-P. Laurent, 1999, Building models for credit spreads, Journal of Derivatives, 6 (3), 27-43.

Bakshi G., D. Madan and F. Zhang, 2001, Investigating the sources of default risk: Lessons from empirically evaluating credit risk models, Working Paper, University of Maryland.

Basle Committee on Banking Supervision of the Bank of International Settlement, 1999a, Credit risk modelling: Current practices and applications, Basle.

Basle Committee on Banking Supervision of the Bank of International Settlement, 1999b, A new capital adequacy framework, Basle.

Black F. and J.C. Cox, 1976, Valuing corporate securities: Some effects of bond indentures, Journal of Finance 31, 351-367.

Brennan, M.J. and E.S. Schwartz, 1977, Convertible bonds: valuation and optimal strategies for call and conversion, Journal of Finance 32, 1699-1715.

Brennan, M.J. and E.S. Schwartz, 1980, Analyzing convertible bon/-ds, Journal of Financial and Quantitative Analysis 15, 907-929.

Brown, S.J. and Ph.H. Dybvig, 1986, The empirical implications of the Cox, Ingersoll, Ross theory of the term structure of interest rates, Journal of Finance 41, 617-630.

Bühler, W., K. Düllmann and M. Windfuhr, 2001, Affine models, credit spreads and the delivery option of a multi-issuer bond future, Working Paper, University of Mannheim.

Campbell, J.Y., A.W. Lo and A.C. MacKinlay, 1997, The econometrics of financial markets (Princeton University Press, Princeton).

Carty, L.V. and D. Lieberman, 1996, Corporate bond defaults and default rates 1938 - 1995, Moody´s Investors Service, Global Credit Research, January 1996.

Chan, K.C., G.A. Karolyi, F.A. Longstaff and A.B. Sanders, 1992, An empirical comparison of alternative models of the short term interest rate, Journal of Finance 47, 1209-1227.

Cooper, I. and M. Martin, 1996, Default risk and derivative products, Applied Mathematical Finance 3, 53-74.

Cooper, I. and A. Mello, 1988, Default spreads in the floating interest rate markets: A contingent claims approach, Advances in Futures and Options Research 3, 269-289.

Coppes R.C., 1997, Credit risk exposure with currency swaps, European Financial Management 3, No. 1, 85-98.

Cossin, D. and T. Hricko, 2001, Exploring for the determinants of credit risk in credit default swap transaction data, Working Paper, HEC, University of Lausanne.

Crabbe, L.E. and C.M. Turner, 1995, Does the liquidity of a debt issue increase with its size? Evidence from the corporate bond and medium-term note markets, Journal of Finance 50, 1719-1735.

Das, S.R. and P. Tufano, 1996, Pricing credit-sensitive debt when interest rates, credit ratings and credit spreads are stochastic, Journal of Financial Engineering 5 (2), 161-198.

De Munnik, J.F.J. and P.C. Schotman, 1994, Cross-sectional versus time series estimation of term structure models: empirical results for the Dutch bond market, Journal of Banking and Finance 18, 997-1025.

Duffee, G.R., 1998, The relation between treasury yields and corporate bond yield spreads, Journal of Finance 53, 2225-2241.

Duffee, G.R., 1999, Estimating the price of default risk, Review of Financial Studies 12, 197-226.

Duffie, D., 1998, Defaultable Term Structure Models with Fractional Recovery of Par, Working Paper, Stanford University.

Duffie D. and N. Garleanu, 2001, Risk and valuation of collateralized debt obligations, Financial Analysts Journal, Jan/Feb, 41-59.

Duffie, D. and M. Huang, 1996, Swap rates and credit quality, Journal of Finance 51, 921-949.

Duffie, D. and D. Lando, 2001, Term structures of credit spreads with incomplete accounting information, Econometrica 69, 633-664.

Duffie, D., L.H. Pedersen and K.J. Singleton, 2001, Modeling sovereign yield spreads: A case study of Russian debt, Working Paper, Stanford University.

Duffie, D. and K.J. Singleton, 1997, An econometric model of the term structure of interest-rate swap yields, Journal of Finance 52, 1287-1321.

Duffie, D. and K.J. Singleton, 1999, Modeling Term Structures of Defaultable Bonds, Review of Financial Studies 12 (4), 687-720.

Düllmann, K., M. Uhrig-Homburg and M. Windfuhr, 1998, Risk structure of interest rates: An empirical analysis for Deutschemark-denominated bonds, Working Paper, University of Mannheim.

Düllmann, K. and M. Windfuhr, 2000, Credit Spreads Between German and Italian Sovereign Bonds - Do One-Factor Affine Models Work?, Canadian Journal of Administrative Sciences 17 (2), 166-181.

Elton E.J., M.J. Gruber, D. Agrawal and C. Mann, 2001, Explaining the rate spread on corporate bonds, Journal of Finance 56, 247-277.

Ericsson, J. and O. Renault, 2001, Liquidity and credit risk, Working Paper, London School of Economics & Political Science.

Estrella, A. and G.A. Hardouvelis, 1991, The term structure as a predictor of real economic activity, Journal of Finance 46, 555-576.

European Shadow Financial Regulatory Committee, 1999, Statement No. 4, Statement No. 5, available under
http://www.aei.org/shdw/european.htm.

Fama E.F., 1990, Term structure forecasts of interest rates, inflation, and real returns, Journal of Monetary Economics 25, 59-76.

Fridson, M.S. and J.G. Jonsson, 1995, Spread vs. treasuries and the riskiness of high-yield bonds, Journal of Fixed Income, 79-88.

Geske, R., 1977, The valuation of corporate liabilities as compound options, Journal of Financial and Quantitative Analysis 12, 541-552.

Grinblatt M., 1995, An analytic solution for interest-rate swap spreads, to appear in: Review of International Finance 2001.

Gupton, G.M., D. Gates and L.V. Carty, 2000, Bank loan loss given default, Moody´s Investors Service, Nov. 2000, available under:
http://riskcalc.moodysrms.com/us/research/

Gupton, G.M., C.C. Finger and M. Bhatia, 1997, CreditMetrics$^{TM}$ - Technical Document, J.P. Morgan & Co., New York, 1997, available under: http://www.riskmetrics.com.

Harvey, C.R., 1991, The term structure and world economic growth, Journal of Fixed Income 1, 7-19.

Harvey, C.R., 1993, Term structure forecasts economic growth, Financial Analysts Journal, May-June, 6-8.

Heath, D.R., R.A. Jarrow and A. Morton, 1992, Bond pricing and the term structure of interest rates: a new methodology for contingent claim valuation, Econometrica 60, 77-105.

Houweling P., J. Hoek and F. Kleibergen, The joint estimation of term structures and credit spreads, 2001, Journal of Empirical Finance 8 (3), 297-323.

Huang J.-Z. and M. Huang, 2000, How much of the corporate - Treasury yield spread is due to credit risk?, Working Paper, Stanford University.

Hull, J. and A. White, 1995, The impact of default risk on the prices of options and other derivative securities, Journal of Banking and Finance 19, 299-322.

International Swaps and Derivatives Association, 1998, Credit risk and regulatory capital, available under
http://www.geocities.com/WallStreet/8589/crr_papers.htm.

Jarrow, R.A., D. Lando and S.M. Turnbull, 1997, A markov model for the term structure of credit risk spreads, The Review of Financial Studies 10, 481-523.

Jarrow, R.A., D. Lando and F. Yu, 2001, Default Risk and Diversification: Theory and Applications, Working Paper, Johnson Graduate School of Management, Cornell University, Ithaca.

Jarrow, R.A. and S.M. Turnbull, 1995, Pricing derivatives on financial securities subject to credit risk, Journal of Finance 50, 53-86.

Jarrow, R.A. and S.M. Turnbull, 2000, The intersection of market and credit risk, Journal of Banking and Finance 24, 271-299.

Jarrow, R.A. and F. Yu, 1999, Counterparty risk and the pricing of defaultable securities, Working Paper, Johnson Graduate School of Management, Cornell University, Ithaca.

Jones, E., S. Mason and E. Rosenfeld, 1984, Contingent claims analysis of corporate capital structures: An empirical investigation, Journal of Finance, 611-625.

Kiesel, R., W. Perraudin and A. Taylor, 2001, The structure of credit risk: spread volatility and ratings transitions, Working Paper, London School of Economics.

Kempf, A. and M. Uhrig-Homburg, 2000, Liquidity and its impact on bond prices, Schmalenbach Business Review 52, 26-44.

Kim, J., K. Ramaswamy and S. Sundaresan, 1993, Does default risk in coupons affect the valuation of corporate bonds? A contingent claims model, Financial Management, Autumn, 117-131.

Lamberton, D. and B. Lapeyre, 1996, Introduction to stochastic calculus applied to finance (Chapman & Hall, London).

Lando, D., 1994, Three essays on contingent claims pricing, PhD thesis, Cornell University.

Lando, D., 1997, Modeling bonds and derivatives with default risk, in: M. Dempster and S. Pliska, eds., Mathematics of Derivative Securities (Cambridge University Press, Cambridge), 369-393.

Lande, D., 1998, On Cox processes and credit risky securities, Review of Derivatives Research 2, 99-120.

Leland, H., 1994, Corporate debt value, bond covenants, and optimal capital structure, Journal of Finance 49, 1213-1252.

Litterman, R. and T. Iben, 1991, Corporate bond valuation and the term structure of credit spreads, The Journal of Portfolio Management, 52-64.

Litterman, R. and J. Scheinkman, 1991, Common factors affecting bond returns, Journal of Fixed Income 1, 54-61.

Liu, J., F.A. Longstaff and R.E. Mandell, 2000, The market price of credit risk - An empirical analysis of interest rate swap spreads, Working Paper, Anderson School at UCLA.

Longstaff, F.A. and E.S. Schwartz, 1995, A simple approach to valuing risky fixed and floating rate debt, Journal of Finance 50, 789-819.

Madan, D.B. and H. Unal, 1998, Pricing the risks of default, Review of Derivatives Research 2, 121-160.

Madan, D.B. and H. Unal, 2000, A two-factor hazard rate model for pricing risky debt and the term structure of credit spreads, Journal of Financial and Quantitative Analysis 35, 43-65.

Merton, R.C., 1974, On the pricing of corporate debt: The risk structure of interest rates, Journal of Finance 29, 449-470.

Mishkin, F.S. and A. Estrella, 1996, The yield curve as a predictor of U.S. recessions, Current Issues in Economics and Finance 2 (7).

Moody´s, 1992, Moody´s special report - Corporate bond defaults and default rates, Moody´s Investors Service, New York.

Moody´s, 2000, Moody´s special comment - Historical default rates of corporate bond issuers 1920-1999, Moody´s Investors Service, New York.

Moody´s, 2001, Moody´s risk calc for private companies - The German model, available under:

http://riskcalc.moodysrms.com/us/research/crm/681431.asp.

Morris, C., R. Neal and D. Rolph, 1999, Credit spreads and interest rates: A cointegration approach, Working Paper, Federal Reserve Bank of Kansas City.

Nandi, S., 1998, Valuation models for default-risky securities: An overview, Economic Review - Federal Reserve Bank of Atlanta, Fourth Quarter 1998, 22-35.

Nielsen, L.T., J. Saa-Requejo and P. Santa-Clara, 1993, Default risk and interest rate risk: The term structure of default spreads, Working paper, INSEAD.

Pearson, N.D. and T.-S. Sun, 1994, Exploiting the conditional density in estimating the term structure: An application to the Cox, Ingersoll, and Ross model, The Journal of Finance 47, 1279-1304.

Prigent, J-L, O. Renault and O. Scaillet, 2001, An empirical investigation into credit spread indices, Journal of Risk, Spring 2001, 27-55.

Roma A. and W. Torous, 1997, The cyclical behavior of interest rates, Journal of Finance 52, 1519-1542.

Sarig, O. and A. Warga, 1989, Some empirical estimates of the risk structure of interest rates, Journal of Finance 44, 1351-1360.

Schönbucher, P.J., 1998, Term structure of defaultable bonds, Review of Derivatives Research 2, 161-192.

Schwaiger, W.S.A. and C. Thym, 1999, Arbitragefreie Bewertung von ausfallgefährdeten Anleihen (Krediten) und Kreditderivaten, in: R. Eller, ed., Handbuch Kreditrisikomodelle und Kreditderivate (Schäffer-Pöschel-Verlag, Stuttgart), 243-266.

Shimko, D., N. Tejima and D. vanDeventer, 1993, The pricing of risky debt when interest rates are stochastic, The Journal of Fixed Income 3, September, 58-66.

Skinner, F.S. and A. Diaz, 2000, On modelling credit risk using arbitrage free models, Working Paper, ISMA Centre, University of Reading.

Stambaugh, R.F., 1988, The information in forward rates - Implications for models of the term structure, Journal of Financial Economics 21, 41-70.

Standard & Poor´s, 2000, Standard & Poor´s special report - Ratings performance 1999, New York.

Warga, A., 1992, Bond returns, liquidity, and missing data, in: Journal of Financial and Quantitative Analysis 27, 605-617.

Table 1: Regression: $\widehat{\lambda}_t = \beta_0 + \beta_1 SIZE + \beta_2 AGE + \beta_3 DUMMY_{HY} + \beta_4 DUMMY_B + \eta_t$

| $\widehat{\beta}_0$ | $\widehat{\beta}_1$ | $\widehat{\beta}_2$ | $\widehat{\beta}_3$ | $\widehat{\beta}_4$ | $R^2$ |
|---|---|---|---|---|---|
| 0.0026 | -5.E-13 | -2.4E-5 | 0.0009 | 0.0003 | 0.9294 |
| (11.555) | (-0.453) | (-0.862) | (10.069) | (18.642) | |

Table 2: Quality of Estimates as a Function of $S$

| | Cross Section | | West LB, $T_1 = 2001$ | | West LB, $T_2 = 2003$ | | West LB, $T_3 = 2006$ | |
|---|---|---|---|---|---|---|---|---|
| S | $SD(\lambda)$ | MSPE | $SD(\lambda)$ | MSPE | $SD(\lambda)$ | MSPE | $SD(\lambda)$ | MSPE |
| 1 | 0.0014 | 0.1107 | 0.0012 | 0.0057 | 0.0013 | 0.0270 | 0.0019 | 0.0656 |
| 7 | 0.0013 | 0.1080 | 0.0009 | 0.0047 | 0.0011 | 0.0208 | 0.0016 | 0.0760 |
| 14 | 0.0012 | 0.1102 | 0.0008 | 0.0046 | 0.0010 | 0.0206 | 0.0015 | 0.0823 |
| 21 | 0.0012 | 0.1131 | 0.0007 | 0.0046 | 0.0010 | 0.0223 | 0.0014 | 0.0887 |
| 28 | 0.0011 | 0.1161 | 0.0007 | 0.0045 | 0.0009 | 0.0235 | 0.0013 | 0.0951 |
| 35 | 0.0010 | 0.1194 | 0.0006 | 0.0045 | 0.0011 | 0.0234 | 0.0011 | 0.1005 |
| $\bar{t}/\Delta$ | – | 0.1609 | – | 0.0058 | – | 0.0503 | – | 0.1565 |

Table 3: Regression: $\widehat{\lambda}_t = \beta_0 + \beta_1 t + \beta_2 R(t, t+1) + \beta_3 (R(t, t+10) - R(t, t+1)) + \beta_4 \varepsilon_t + \eta_t$

| $\widehat{\beta}_0$ | $\widehat{\beta}_1$ | $\widehat{\beta}_2$ | $\widehat{\beta}_3$ | $\widehat{\beta}_4$ | $R^2$ |
|---|---|---|---|---|---|
| Cross-Section, S=7 | | | | | |
| 0.0091 | 0.0011 | -0.0069 | -0.2106 | 0.0051 | 0.7057 |
| (4.947) | (1.465) | (-0.123) | (-5.306) | (4.018) | |
| West-LB, $T_1 = 2001$, S=28 | | | | | |
| 0.0012 | -0.0005 | 0.1142 | 0.0345 | -0.0002 | 0.4146 |
| (0.901) | (-0.867) | (2.700) | (1.466) | (-1.560) | |
| West-LB, $T_2 = 2003$, S=14 | | | | | |
| 0.0089 | 0.0005 | -0.0401 | -0.2585 | 0.0011 | 0.7365 |
| (5.714) | (0.816) | (-0.886) | (-7.040) | (8.459) | |
| West-LB, $T_3 = 2006$, S=1 | | | | | |
| 0.0076 | 0.0025 | -0.0218 | -0.1541 | 0.0043 | 0.4268 |
| (1.899) | (1.341) | (-0.168) | (-2.201) | (1.969) | |

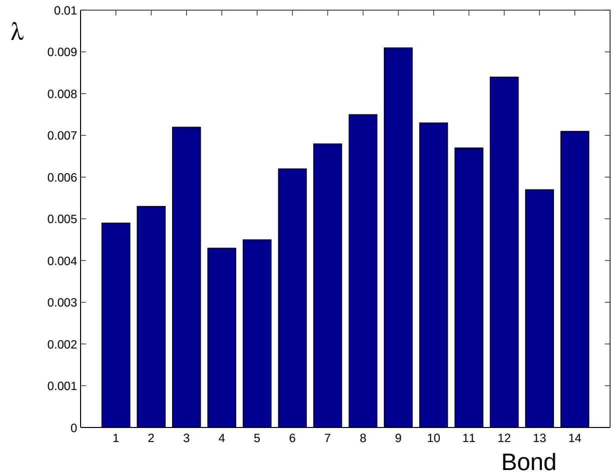


Figure 1: Default intensities from separate estimation, $S = \bar{t}/\Delta$.

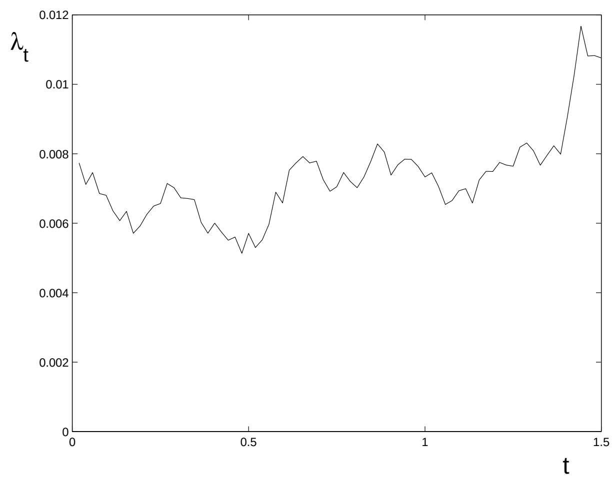


Figure 2: Cross-sectional estimation, $S = 7$.